Aufg 6

Q1

a)

$$(1) \rightarrow \quad 3y = x - \dot{x}$$

$$3\dot{y} = \dot{x} - \ddot{x}$$

$$\dot{y} = \tfrac{1}{3}\dot{x} - \tfrac{1}{3}\ddot{x}$$

einsetzen in (2)

$$\tfrac{1}{3}\dot{x} - \tfrac{1}{3}\ddot{x} = \tfrac{7}{4}x + x - \dot{x}$$

$$-\tfrac{1}{3}\ddot{x} + \tfrac{4}{3}\dot{x} - \tfrac{5}{4}x = 0$$

$$\ddot{x} - 4\dot{x} + \tfrac{15}{4}x = 0$$

$$r^2 - 4r + \tfrac{15}{4} = 0$$

$$r_{1/2} = 2 \pm \sqrt{\tfrac{16}{4} - \tfrac{15}{4}}$$

$$= 2 \pm \tfrac{1}{2} \qquad r_1 = 1.5 \quad r_2 = 2.5$$

$$\Rightarrow \boxed{x(t) = Ae^{1.5t} + Be^{2.5t}}$$

$$\dot{x}(t) = 1.5Ae^{1.5t} + 2.5Be^{2.5t}$$

$$3y = x - \dot{x}$$

$$= -0.5Ae^{1.5t} - 1.5Be^{2.5t}$$

$$\boxed{y(t) = -\tfrac{1}{6}Ae^{1.5t} - \tfrac{1}{2}Be^{2.5t}}$$

alternatively

$$(2) \Rightarrow 0.25x = -3y + \dot{y}$$

$$x = -12y + 4\dot{y}$$

$$\dot{x} = -12\dot{y} + 4\ddot{y}$$

plug into (1)

$$-12\dot{y} + 4\ddot{y} = -12y + 4\dot{y} - 3y$$

$$4\ddot{y} - 16\dot{y} + 45y = 0$$

$$\ddot{y} - 4\dot{y} + \frac{45}{4}y = 0$$

$$\Rightarrow \boxed{y(t) = \tilde{A}e^{1.5t} + \tilde{B}e^{2.5t}}$$

$$\dot{y} = 1.5\tilde{A}e^{1.5t} + 2.5\tilde{B}e^{2.5t}$$

$$\dot{x} = -12\tilde{A}e^{1.5t} - 12\tilde{B}e^{2.5t} + 6\tilde{A}e^{1.5t} + 10\tilde{B}e^{2.5t}$$

$$\boxed{x = -6\tilde{A}e^{1.5t} - 2\tilde{B}e^{2.5t}}$$

$$\left(\text{ie } \tilde{A} = -\tfrac{1}{6}A, \quad \tilde{B} = -\tfrac{1}{2}B \right)$$

b)

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 1/4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{vmatrix} 1-\lambda & -3 \\ 1/4 & 3-\lambda \end{vmatrix} = 3 - 3\lambda - \lambda + \lambda^2 + 3/4$$

$$\Rightarrow \quad \lambda^2 - 4\lambda + \frac{15}{4} = 0$$

$$\lambda_{1/2} = 2 \pm 1/2 \qquad \lambda_1 = 1.5 \qquad \lambda_2 = 2.5$$

eigenvectors

1.5 :

$$u - 3v = 1.5u$$

$$1/4\,u + 3v = 1.5v$$

$$0.5u = -3v$$

$$u = -6v \quad \Rightarrow \begin{pmatrix} 1 \\ -1/6 \end{pmatrix}$$

2.5 :

$$u - 3v = 2.5u$$

$$1.5u = -3v$$

$$u = -2v \quad \Rightarrow \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = Ae^{1.5t} \begin{pmatrix} 1 \\ -1/6 \end{pmatrix} + Be^{2.5t} \begin{pmatrix} 1 \\ -1/2 \end{pmatrix}$$

$$x = Ae^{1.5t} + Be^{2.5t}$$

$$y = -1/6\,Ae^{1.5t} - \frac{1}{2}Be^{2.5t}$$

Q2

a) $(1) \Rightarrow 5y = \dot{x} - x - 18$

$$y = \tfrac{1}{5}\dot{x} - \tfrac{1}{5}x - \tfrac{18}{5}$$

$$\dot{y} = \tfrac{1}{5}\ddot{x} - \tfrac{1}{5}\dot{x}$$

plug into (2)

$$\tfrac{1}{5}\ddot{x} - \tfrac{1}{5}\dot{x} = \tfrac{1}{4}x - \tfrac{1}{5}\dot{x} + \tfrac{1}{5}x + \tfrac{18}{5} + 9$$

$$\tfrac{1}{5}\ddot{x} - \tfrac{9}{20}x = \tfrac{18}{5} + \tfrac{45}{5} = \tfrac{63}{5}$$

$$\ddot{x} - \tfrac{9}{4}x = 63$$

solution of homogeneous

$$r^2 - \tfrac{9}{4} = 0$$

$$r_{1,2} = \pm \tfrac{3}{2}$$

$$x = Ae^{3/2t} + Be^{-3/2t}$$

particular solution

$$-\tfrac{9}{4}x = 63$$

$$x = -\frac{63 \cdot 4}{9} = -7 \cdot 4 = -28$$

$$\Rightarrow x(t) = Ae^{3/2t} + Be^{-3/2t} - 28$$

$$\dot{x} = \tfrac{3}{2}Ae^{3/2t} - \tfrac{3}{2}Be^{-3/2t}$$

$$y = \tfrac{3}{10}Ae^{3/2t} - \tfrac{3}{10}Be^{-3/2t}$$

$$- \tfrac{1}{5}Ae^{3/2t} - \tfrac{1}{5}Be^{-3/2t} + \tfrac{28}{} - \tfrac{18}{5}$$

$\Rightarrow y(t) = \frac{1}{10} A e^{3/2 t} - \frac{1}{2} B e^{-3/2 t} + 2$

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b)

$x(0) = A + B - 28 = 6$

$y(0) = \frac{1}{10} A - \frac{1}{2} B + 2 = 3$

$\Rightarrow A = 34 - B$

$\frac{34}{10} - \frac{B}{10} - \frac{1}{2} B + 2 = 3$

$\frac{54}{10} = \frac{6}{10} B \qquad B = \frac{54}{6} = 9$

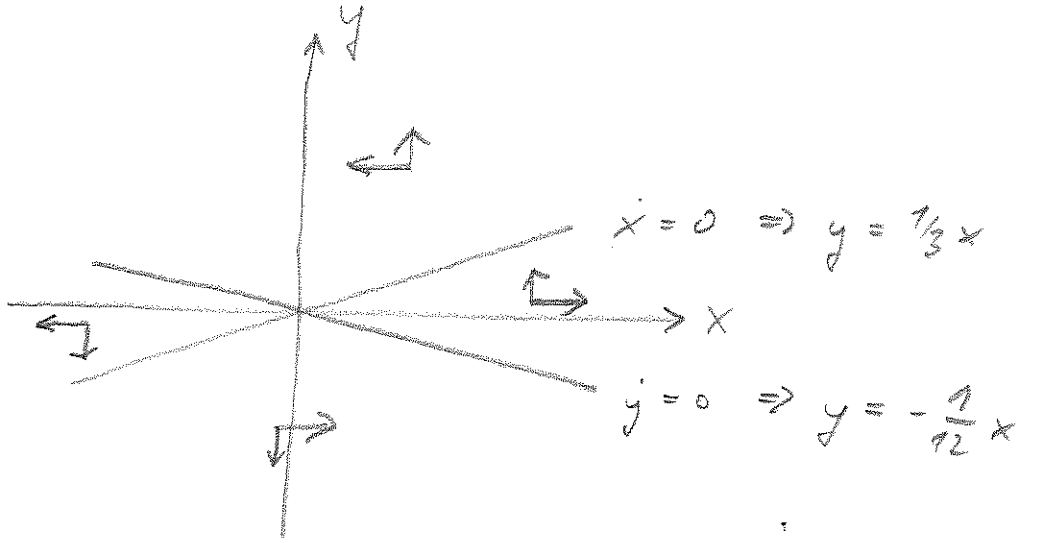
$A = 25$

$x = 25 e^{3/2 t} + 9 e^{-3/2 t} - 28$

$y = 2,5 e^{3/2 t} - 4,5 e^{-3/2 t} + 2$

Q3

1. a)



b) $(x=0, y=0)$

c) $\lambda_1 = 1.5 \qquad \lambda_2 = 2.5$

both positive $\Rightarrow$ explodes

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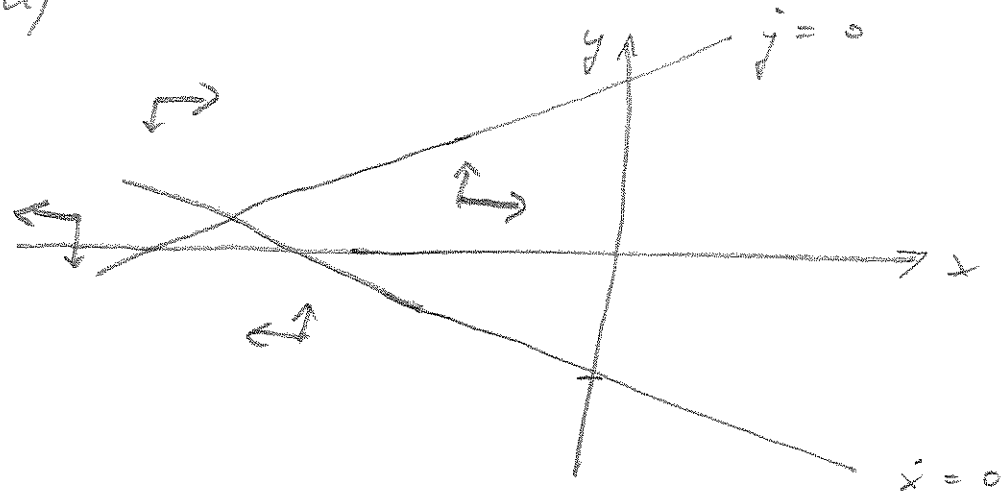
2. $\dot{x}=0 \quad 5y=-x-18 \quad y=-\frac{1}{5}x-\frac{18}{5}$

$\dot{y}=0 \quad y=\frac{1}{4}x+9$

a)



b) $-\frac{1}{5}x-\frac{18}{5}=\frac{1}{4}x+9$

$-\frac{9}{20}x=\frac{45}{5}+\frac{18}{5}=\frac{63}{5}$

$x=-\frac{20\cdot 63}{5\cdot 9}=-\frac{4\cdot 7}{1}=-28$

$y = \frac{1}{4}(-28) + 9 = -7 + 9 = 2$

$\Rightarrow \quad (-28, 2)$

c)

$$\begin{vmatrix} -1-\lambda & 5 \\ 1/4 & -1-\lambda \end{vmatrix} = -1 - \lambda + \lambda + \lambda^2 - 5/4 = 0$$

$$\lambda^2 - 9/4 = 0$$

$$\lambda_{1/2} = \pm \frac{3}{2}$$

$\Rightarrow$ saddle path