

## Assignment 8

due: Thursday, May 10, in class

**Problem 1:** Consider  $\max \int_0^T (xu - u^2 - x^2)dt$  with  $\dot{x} = u$  and  $x(0) = x_0$ .

- Write down the Hamiltonian and the necessary conditions.
- Solve for  $p(t)$ ,  $u(t)$ , and  $x(t)$ , using the initial and the transversality condition.
- Check the sufficient condition.

**Problem 2:** Maximize  $\int_0^1 (-2\dot{x} - \dot{x}^2)e^{-t/10}dt$  with  $x(0) = 1$  and  $x(1) = 0$ .

- Solve by using the Euler equation.
- Solve by using the Hamiltonian.
- Solve by using the current value Hamiltonian.

**Problem 3:** A firm's production function is  $Y = f(K) = K - aK^2$ , its capital stock evolves according to  $\dot{K} = I - \delta K$ , where  $I$  is its investment and  $K(0) = K_0$  is given, and its objective is to maximize the stream of profits  $\int_0^T (K - aK^2 - I^2)dt$ . Find its optimal investment behavior and the resulting capital stock over time.