Final exam (part 1)

This is a 2 hour exam. There are three equally weighted questions. Please read them carefully and answer *each* of them. If you have problems understanding a question please do ask. *Good luck !!!*

Question 1: Consider an exchange economy populated by Angela and Nicolas. Both of them have identical preferences which can be expressed by $U(x_1, x_2) = x_1 x_2$, take prices as given, and have endowment vectors $(e_1^A = 8, e_2^A = 0)$ and $(e_1^N = 0, e_2^N = 8)$ respectively.

a) Draw the corresponding Edgeworth box (pls label the axes and origins), indicate the endowment point, find the specific contract curve, and depict it in the Edgeworth-box.

b) Calculate the market outcome, i.e. the equilibrium price and qunatities. Does the equilibrium allocation lie on the contract curve? Does it have to?

c) Suppose there is a transaction tax of t percent whenever Angela and Nicolas exchange goods, and that the government simply disposes of the revenue. Depict the resulting equilibrium in an Edgeworth box. How does the size of this box compare to the one you drew under a)?

Question 2: Consider a pure exchange economy in which I consumers (indexed $i = 1, ..., I$) trade $K + 1$ goods (indexed $k = 0, ..., K$). Let good 0 serve as the numeraire, and take its price to be equal to unity. Suppose that the utility of the i-th consumer is given by

$$u_i(x_i) = x_{i0} - \sum_{k=1}^{K} \alpha_{ik} \frac{e^{-\beta x_{ik}}}{\beta},$$

where $\alpha_{ik}, \beta > 0$. To keep things simple, assume that negative values of x_0 are feasible. Let $\omega_i = (\omega_{i0}, ..., \omega_{iK})$ denote the initial endowment of consumer i.

a) Solve explicitly for the aggregate excess demand function $\xi_k(p)$ for each good $k = 1, ..., K$ as a function of the price vector p.

b) Solve explicitly for a competitive equilibrium price vector. Show and explain how prices change with changes in α_{ik}, ω_{ik}, and β.

c) Let $\xi(p)$ denote the aggregate excess demand vector for goods $k = 1, ..., K$. Compute $\det D\xi_p$. What can you deduce about the uniqueness of competitive equilibrium from this determinant? If the equilibrium is unique, to what feature of the demand functions is this traceable?

Question 3: Consider the framework used in section 16.F of MWG.

a) Explicitly derive the three types of conditions (16.F.4–16.F.6) for Pareto Optimality.

b) Choose appropriate prices and show that the respective optimality conditions of consumers and producers (16.F.10 and 16.F.11) correspond to the conditions you derived under a).

c) Standard definitions of price equilibria (e.g. Def 16.B.3) not only require consumption and production optimality, but also market clearing. Are conditions 16.F.10 and 16.F.11 sufficient for market clearing?