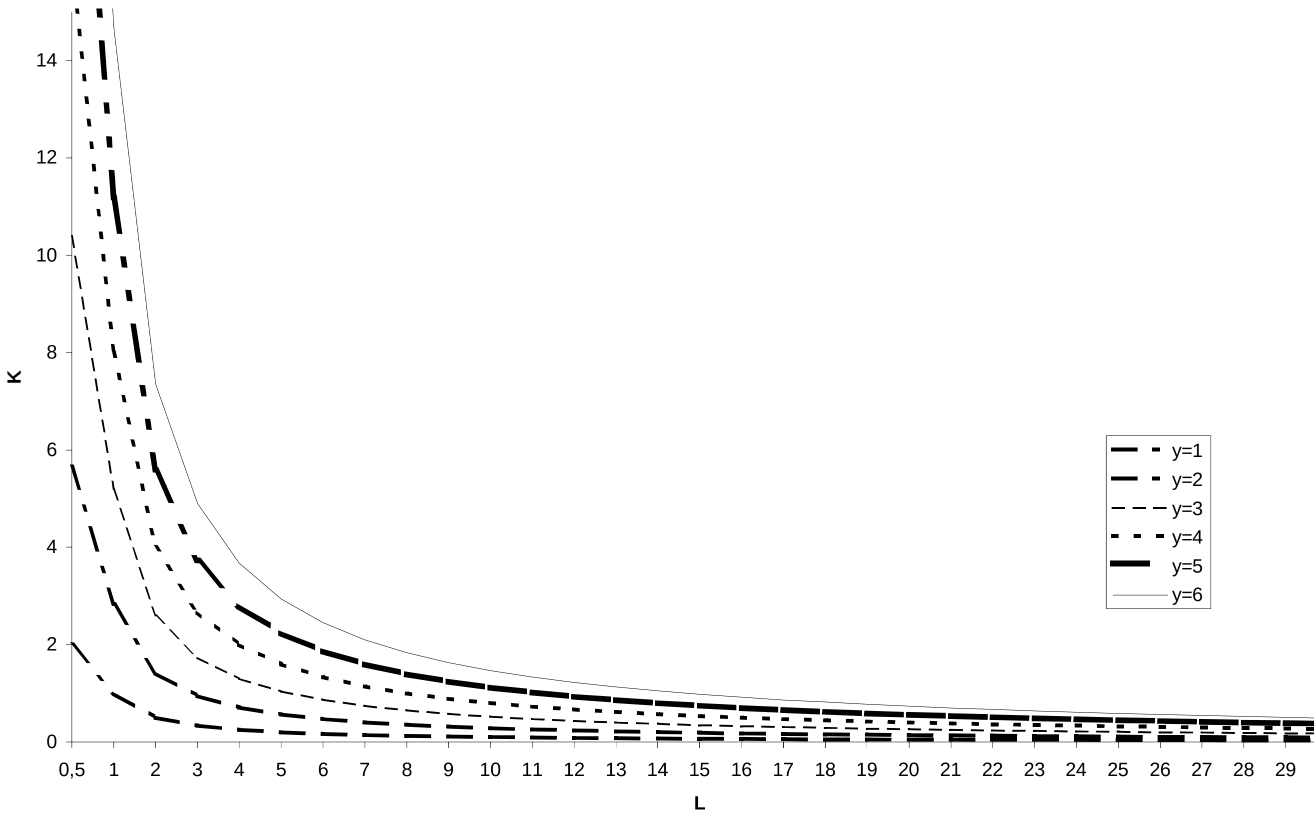
Handout 1: Hints

Exercise 1

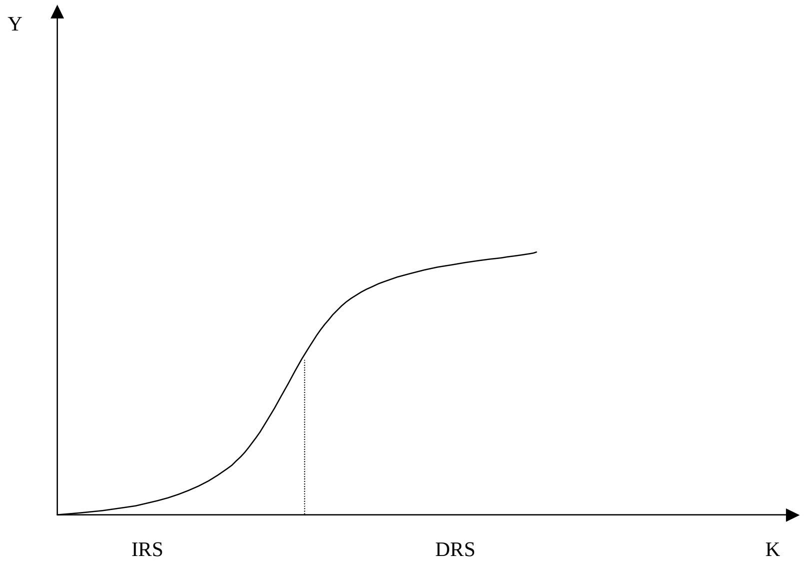
$F(K,L) = K^{2/3}L^{2/3}$



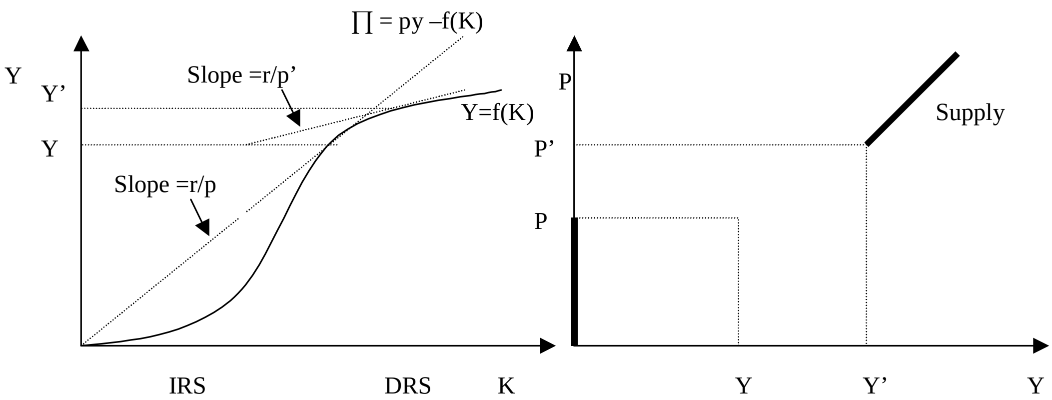
- $F(K,L) = K^{2/3}L^{2/3}$; $F(\theta K,\theta L) = (\theta K)^{2/3}(\theta L)^{2/3} = \theta^{4/3}F(.)$; Increasing returns to scale; (TIPPS: cobb douglas with $\alpha + \beta > 1$)
- It exhibits increasing return to scale because the distance between each Isoquant is smaller and smaller.
- $\begin{cases} MP_k = \dfrac{\partial F}{\partial K} = \dfrac{2}{3}K^{-1/3}L^{2/3} \\ \dfrac{\partial MP_k}{\partial K} < 0 \end{cases} \Rightarrow decrea\sin g$ MP; Do the same for L
- $MRST_{k,l} = \dfrac{MP_k}{MP_l} = \dfrac{L}{K}$
- A production function can have diminishing marginal product in each factor and still exhibits increasing returns to scale. (See the definitions)

Exercise 2

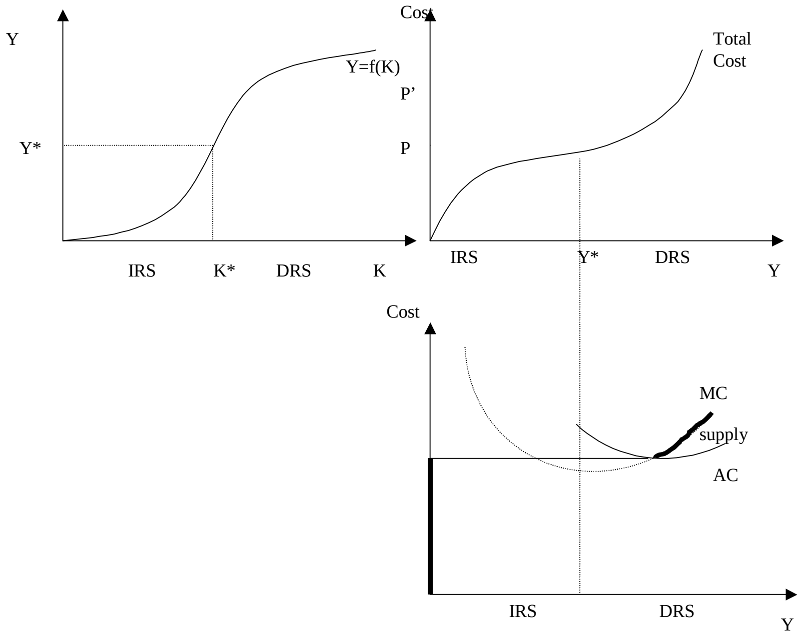
a)



b) supply of Y given $w = \bar{w}$



c) d) e)



f) The two supply curve are identical. the represent the output for a given of a profit maximizing price taker

Exercise 3

$Y = F(K,L) = K^{\alpha}L^{\beta}$

a.

i. show for which condtions that $\frac{\partial MP_k}{\partial K}$ and $\frac{\partial MP_l}{\partial L}$ is negative

ii. $Y = F(\theta K, \theta L) = (\theta K)^{\alpha}(\theta L)^{\beta} = \theta^{\alpha+\beta}F(K,L)$

The production function is homgenous of degree $\alpha + \beta$

$\alpha + \beta > 1 \Rightarrow increa\sin g\, returns$

$\alpha + \beta < 1 \Rightarrow decrea\sin g\, returns$

$\alpha + \beta = 1 \Rightarrow cons\tan t\, returns$

b. $\alpha + \beta = 1$

$$\begin{cases} Min\, rK + wL \\ s.t\, \bar{Y} = K^{\alpha}L^{\beta} \end{cases} \Rightarrow Set\, the Lagrangian$$

$£ = rK + wL + \lambda(\bar{Y} - K^{\alpha}L)$

$$\Rightarrow \begin{cases} (1)\ \frac{\partial £}{\partial K} = r - \lambda \alpha K^{\alpha-1} L^{1-\alpha} = 0 \\ (2)\ \frac{\partial £}{\partial K} = w - \lambda(1-\alpha) K^{\alpha} L^{-\alpha} = 0 \\ (3)\ \frac{\partial £}{\partial \lambda} = \bar{Y} - K^{\alpha} L^{\beta} = 0 \end{cases}$$

$$\Rightarrow Plug\,(1)\,and\,(2) \Rightarrow (3')$$

$$\Rightarrow (3')\,in\,(3) \Rightarrow \begin{cases} K = \bar{Y}.\,(\frac{1-\alpha}{\alpha}\frac{r}{w})^{\alpha-1} \\ L = \bar{Y}.\,(\frac{1-\alpha}{\alpha}\frac{r}{w})^{\alpha} \end{cases}$$

$$\Rightarrow C(r,w,y) = r^{\alpha+1}\,(\frac{1-\alpha}{\alpha}\frac{1}{w})^{\alpha}.\bar{Y} + w^{-\alpha}\,(\frac{1-\alpha}{\alpha}r)^{\alpha-1}.\bar{Y}$$

c. Assume $\alpha + \beta < 1$ (in order to get an output profit maximizing outcome)
Solve the following for y

$$\underset{y}{Max}\ \Pi = pY - C(r,w,y)$$

We know C(.) and the production function. Moreover there is decreasing return to scale such that the solution will be

$$p - C'(Y) = 0$$

derive Y(r,w,p) from this solution and plug it into the conditional input demand from b. and get the unconditional input demand:

$$K(r,w,Y(r,w,p)) = K(r,w,p)$$

d. Solve the following for K and L

$$\begin{cases} \underset{K,L}{Max}\ \Pi = pY - rK - wL \\ s.t\,Y = F(K,L) \end{cases}$$

obtain the unconditional inputs demand $K(r,w,p)$ and $L(r,w,p)$. Plug them to the Production function and obtain $Y(r,w,p)$ the profit maximising supply

e. Under IRS, each additional unit of output contributes to a larger profit margin. There is no profit maximizing output.