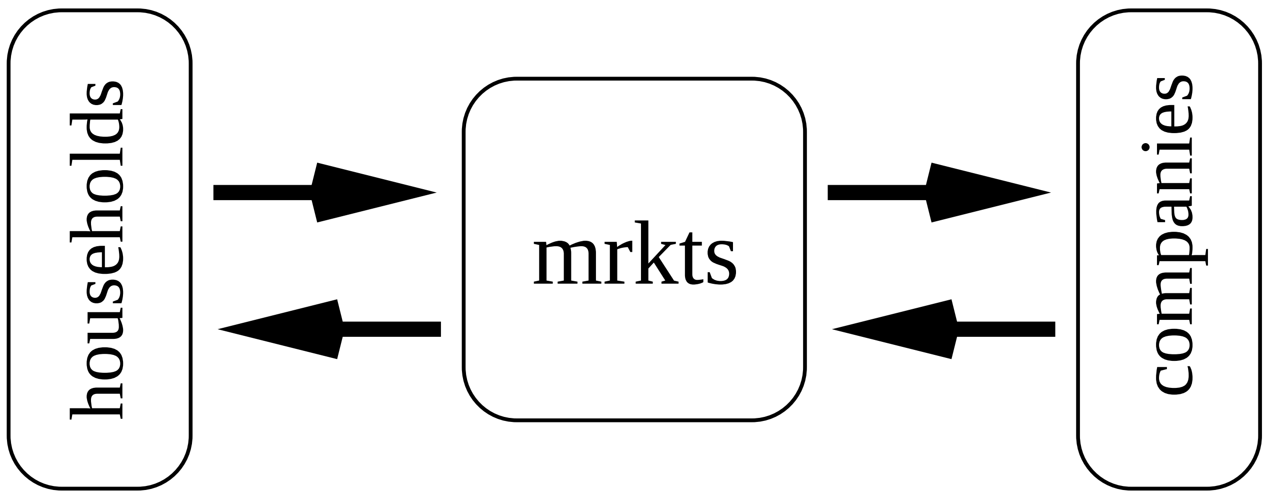
Lecture Notes 5: General Equilibrium

We are now ready to analyze the equilibrium in *all* markets, that is why this type of analysis is called general equilibrium analysis. Recall our graphical overview of a complete market economy:



Our main objective is to find the market equilibrium and investigate its properties. Such an equilibrium in perfectly competitive markets is also called a competitive equilibrium. Similar to partial equilibria, it is a combination of price and quantity, only of higher dimension.

A price vector and the corresponding quantities supplied and demanded are an equilibrium if the following conditions are satisfied:

- given the price vector and constrained by the resulting budget constraints, the respective quantities are utility maximizing for each and every consumer.

- given the price vector and constrained by their technologies, the respective quantities are profit maximizing for each and every firm.

- demand equals supply in each and every market.

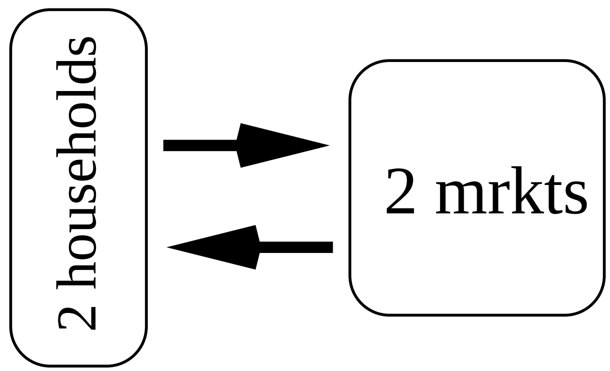
Although it is possible to find and investigate such equilibria for economies with many producers/goods/markets and many consumers, we refrain from doing so because it would require more mathematics than we are willing to use.

Instead we focus on two special cases which between them will highlight all important aspects:

- exchange economies (only households, no production — just endowments)
- Robinson Crusoe (one consumer + one producer)

Exchange Economies

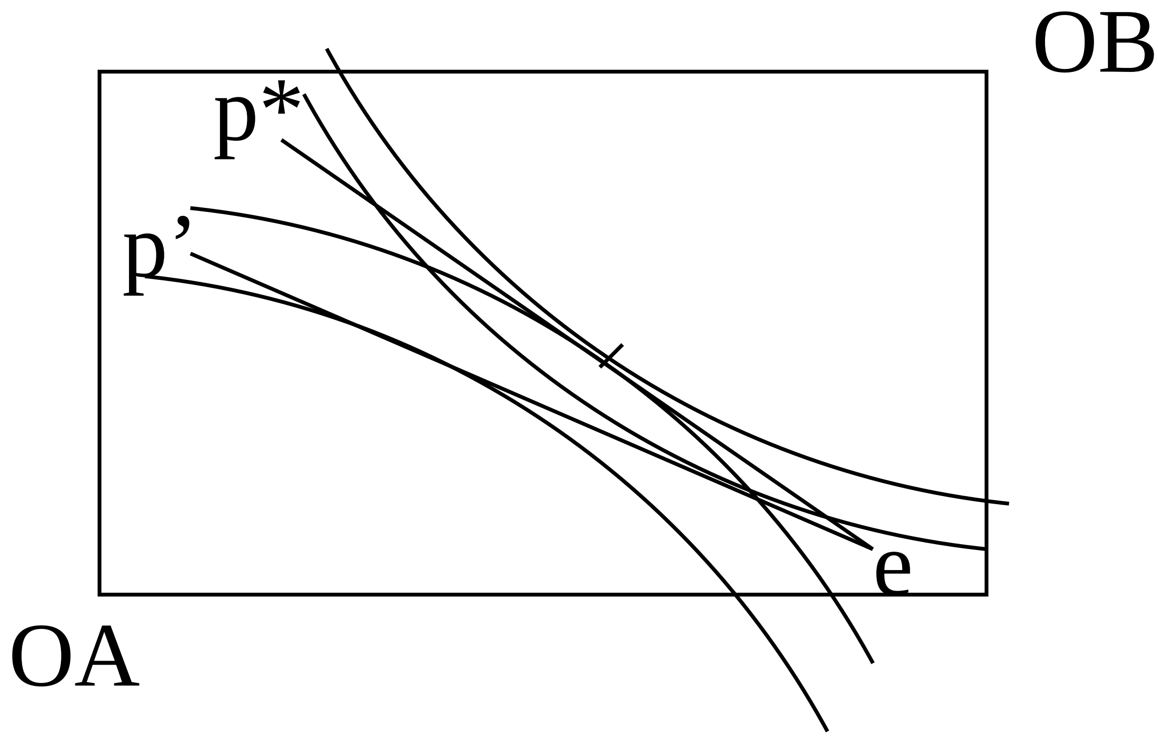
Exchange economies are economies without production. Consumers simply have endowments which they exchange, i.e. sell and buy to/from each other.



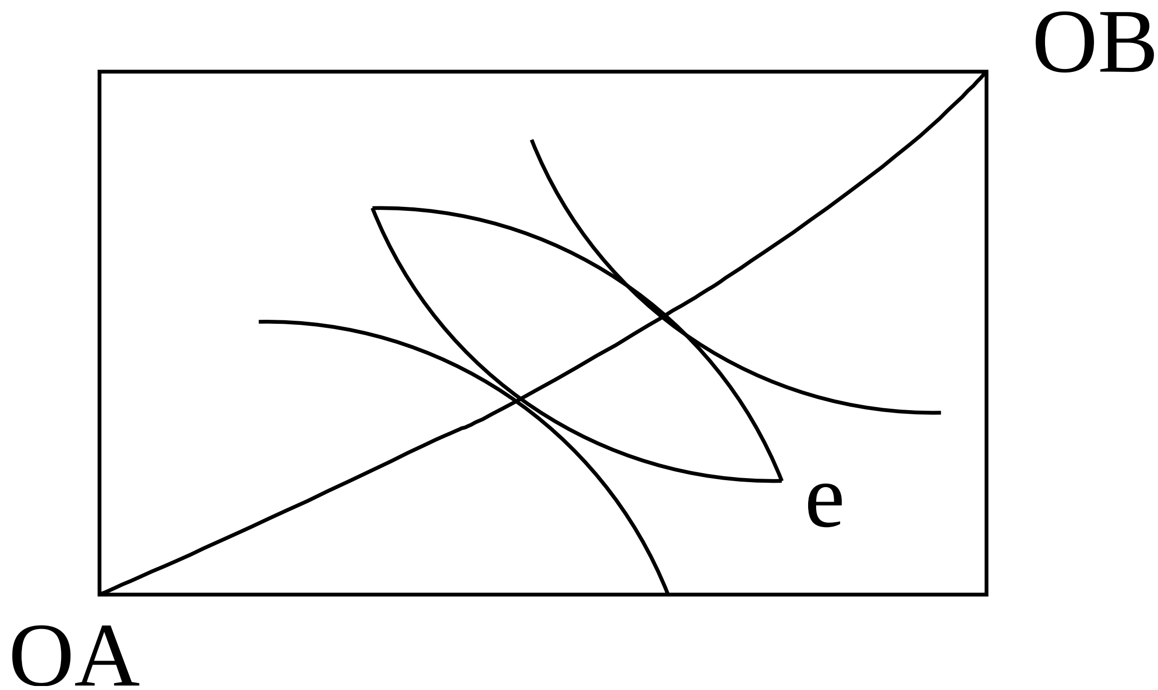
So production is left out of the picture and we focus on the interaction between consumers in this special case.

To make things even simpler, we will mostly consider only two goods/markets and only two consumers. Note, however, that these two markets are all markets in our (admittedly simple) economy and that the two consumers are assumed to be price takers. If there were really only two market participants they would have market power, of course, but keep in mind that we only limit their number to two to make life simple and graphical illustration possible.

The diagram below is called an **Edgeworth box** after the economist who invented it. Note that the size of the box is determined by the aggregate endowments and the SW and NE corners are the origins for the two consumers — one from the Northern hemisphere and the other from down under. The special achievement of this two-dimensional box is that every point inside the box determines 4 quantities: 2 for each consumer.



Starting at the endowment point e, how do we actually find the market equilibrium? Let us try a price p' and see how much consumers A and B demand/supply at that price. Since their optimal (tangency) points do not coincide there must be excess demand in one market and excess supply in the other. The relative price will change accordingly, rotating the trading line (or budget constraint) clockwise. A market equilibrium will be a relative price such as $p*$ where those two optimal tangency points coincide and therefore $MRS^A =$ relative price $= MRS^B$.



Now, let us leave markets and prices aside for a few minutes. Consider the ellipse between the two indifference curves passing thru e. This is the set of mutually beneficial trades or allocations both consumers prefer to e. Note that the market equilibrium must be one of those mutually beneficial points. If it were not then (at least) one consumer would prefer to keep e and refuse to trade.

Consider the set of points that are not dominated by mutually preferred alternatives. This set or curve coincides with all the tangency points of A's and B's indifference curves (except where it hits the boundary and the corresponding ellipse would lie outside the box). This curve is called the **contract curve** because if A and B were free to sign a contract they would always choose a point on the contract curve. All of these points have a certain optimality property.

You have probably heard of the Pareto criterion: an allocation is **Pareto optimal** or efficient iff there is no alternative that leaves no-one worse off but makes at least one person better off. The contract curve is the set of Pareto efficient points in this economy.

But the market equilibrium will also be one of those points. We saw above that if each consumer acts optimally her MRS must equal the (relative) market price. But since both of them face the same price, it must be the case that in equilibrium their respective MRSs are equal. And therefore the market equilibrium lies on the contract curve and must be Pareto efficient.

This illustrates what is called the **first welfare theorem**: Under rather general assumptions any competitve market equilibrium is Pareto efficient.

Note that Pareto efficiency is no guarantee of social desirability. Mr. Gates having everything and everyone else nothing could well be Pareto efficient because there would be no alternative that improves life for others without hurting Mr. Gates.

Let us now turn to Walras' law, another general result that holds in GE (general equilibrium) models and which we will demonstrate using the special case of an exchange economy.

Walras' law: if demand equals supply in all but one market, i.e. in (n-1) markets, then demand must equal supply also in the nth market.

To make the argument consider the following matrix:

$$\begin{matrix} p_1(x_1^A - e_1^A) & p_2(x_2^A - e_2^A) \\ p_1(x_1^B - e_1^B) & p_2(x_2^B - e_2^B) \end{matrix}$$

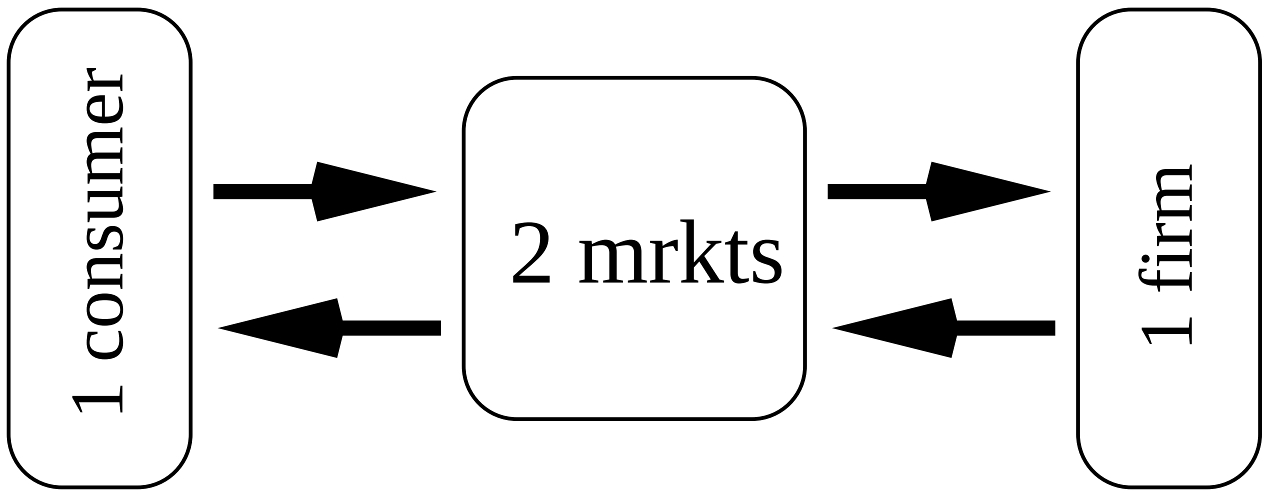
Let us first consider the rows: summing the two elements in each row and setting those sums equal to zero represents consumer A's (first row) and B's (2nd row) budget constraints. Now turn to the columns: summing the two elements in each column and setting the sum equal to zero represents market clearing in market one (column 1) and market 2 (column 2) respectively (feel free to divide away the prices if that makes things clearer).

What Walras law, applied to this special case, claims is that if we have demand equal supply in one market then the same must be true for the other market. Now, we know that the whole matrix sums to zero because budget constraints must be satisfied so each row is zero. But then if one column is zero (supply = demand in one market) the same must be true for the other market because otherwise the whole matrix could not sum to zero.

Note one important implication of Walras' law: To find the equilibrium price vector (of n prices) we have demand = supply in n markets, ie n equations. But Walras' law tells us that one of these n equations is redundant (b/c it is satisfied automatically) and only (n-1) equations contain information. But (n-1) equations are not enough to determine n prices. Instead they only allow us to determine (n-1) relative prices. The price level itself is not determined, in other words inflation does not matter, and we have one degree of freedom in the price vector. In other words, if (2, 3, 4) is an equilibrium price vector then so is (3, 4.5, 6) and any other vector preserving the same relative prices.

Robinson Crusoe

Let us consider the second special case and forget its name for a second.



We focus on the interplay of one producer and one consumer in two markets. Again, these two markets are all markets of our economy. Furthermore, we assume our single producer as well as our single consumer to be price takers despite them being the only seller/buyer of their respective wares. Recall that we limited numbers only to make life easier but that we are really interested in an economy with many producers/consumers where this assumption is (more) realistic.

The one producing company has to belong to someone and since there is only one consumer he/she must be the owner of that company and receive its profits. But then the owner is the only economic agent in this simple model selling/buying things to/from himself and that's why this example is named after Robinson Crusoe. Note though, that Robinson must be somewhat schizophrenic to play the double role of price-taking producer and consumer interacting only with his alter ego.

Let the two goods/markets be coconuts c and labor l. Then Robinson the consumer solves the following utility optimization:

$$\max_{c,l} U(c,l) \quad \text{s.t.} \quad pc = wl + \pi$$

The FOCs can be summarized as $MRS = w/p$ and solving this equation and the budget constraint for c and l gives Robinson's coconut demand function $c^D(p,w,\pi)$ and his labor supply function $l^S(p,w,\pi)$.

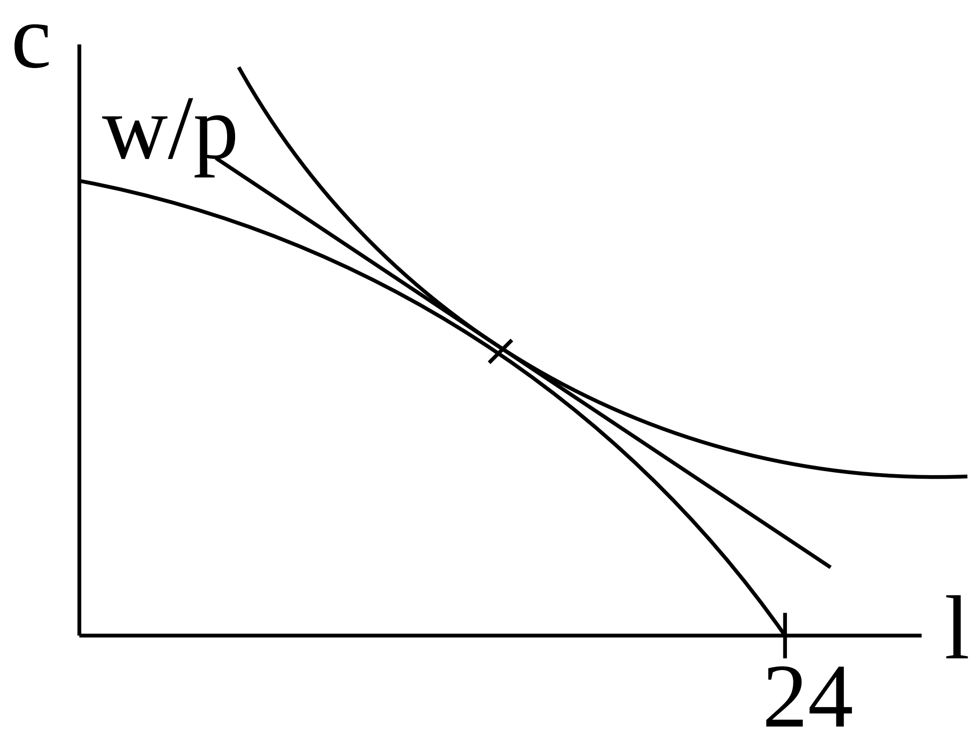
Let us turn to his alter ego, the coconut producer Crusoe Incorporated. This company faces the following profit maximization problem:

$$\max_{c,l} \pi = pc - wl \quad \text{s.t.} \quad c = f(l)$$

Substituting in the production function leads to the FOC: $\delta f / \delta l = w/p$.

A sidenote: you remember the MRTS, the marginal rate of technical substitution between one input and another. There is also the MRT, the marginal rate of transformation (between one output and another), once we have multiproduct firms. The MP of labor on the LHS of the above FOC is something in between: the marginal rate of transforming input into output. In GE there is a tradition of not distinguishing between inputs and outputs. After all, this distinction is rather artificial because many goods are outputs as well as inputs. But then, in fully general GE, there is no difference between MRTS and MRT and the above FOC can be summarized as $MRT(S) = w/p$.

Solving it for l gives Crusoe Inc.'s labor demand function, $l^D(p, w)$. Plugging this into the production function one obtains the coconut supply function $c^S(p, w)$. Finally, plugging both into the definition of profit results in the (maximized) profit function $\pi(p, w)$.



A market equilibrium in this (simple) economy is the price plus the corresponding quantities which clear both markets. Mathematically, it is the price vector (p^*, w^*) that equates $l^D(p, w) = l^S(p, w, \pi(p, w))$ and $c^D(p, w, \pi(p, w)) = c^S(p, w)$. Note that we have plugged in the profit function because Robinson owns Crusoe Inc. and we cannot have exogenous income in a model that represents the entire economy.

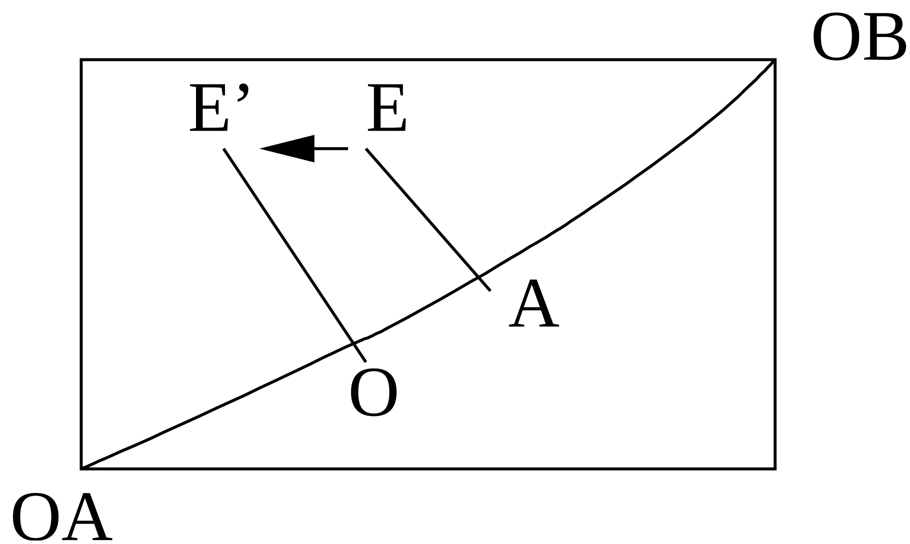
But can we really solve for (p^*, w^*)? What about Walras' law? Suppose the market for coconuts is in equilibrium, i.e. $c^S = c^D$ or $pc^S = pc^D$. On the production side profit is simply $\pi = pc^S - wl^D$ or $pc^S = \pi + wl^D$. Robinson has to satisfy his budget constraint so $pc^D = wl^S + \pi$. It follows that the labor market must be in equilibrium as Walras' law predicts because $\pi + wl^D = wl^S + \pi$ implies $l^D = l^S$. You can easily reverse the order of the argument to show that equilibrium in the labor market implies equilibrium for coconuts. So we cannot solve for an equilibrium price vector but only for $(n - 1)$ relative price(s), i.e. the relative price or real wage w/p.

Now, suppose Robinson Crusoe somehow gets over his schizophrenia and stops playing price-taking producer and consumer interacting with himself in markets. Instead he simply chooses the optimal point (individual central planning in some sense). Mathematically, his optimal point is the solution to $\max U(c,l)\ s.t.\ c = f(l)$. Plugging the contraint into the objective or setting up the Lagrangean give rise to the following FOC: $MRS = MRT$. His optimum happens to be the same tangency point as before. But this time a healthy Robinson simply picks it without any need for prices or markets.

This demonstrates yet another general result: Note first that Robinson's optimum is Pareto efficient (pls check the definition) even though Pareto efficiency looses some of its flavor when applied to a one-person economy. What we see is that this Pareto efficient allocation can be achieved (or decentralized) as the outcome of a market process.

This a manifestation of the **second welfare** theorem: under conditions that guarantee the existence of a market equilibrium, any Pareto optimum can be decentralized as a market equilibrium provided endowments are distributed apropriately.

To understand the last qualification, return to the earlier example of an exchange economy:



If we want to achieve a Pareto efficient point such as O then the second welfare theorem claims that this can be decentralized as the outcome of a market economy. But starting at the endowment point E we arrive at A which is also a Pareto optimum as the first welfare theorem predicts but not the one we wanted to achieve. To achieve O we first need to redistribute endowments to start at point E' and then let the market work its magic.