Trade, Education, and The Shrinking Middle Class*

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Abstract

This paper studies how trade and educational institutions interact to shape the distribution of human capital, both within and across countries. We exploit the multiplicity of sectors and continuous support of possible human capital choices in our framework to demonstrate that freer trade can induce crowding out of the middle occupations towards the skill acquisition extremes in one country and simultaneous expansion of middle-income industries in another. Individual gains from trade may be non-monotonic in ability type, and middle ability agents can lose the most from trade liberalization. Endogenizing trade and education policy, we find that targeted education subsidies are more effective than tariffs as a means to preserve "middle class" jobs, while uniform educational subsidies are of little consequence.

Keywords: Trade and Education Policy, Skill Acquisition, Education, Income Distribution

JEL Classifications: F11, F13, F15, F16

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1 Introduction

Politicians tend to portray education as a universal panacea for rising income inequality and perceived competition with foreign exporters, yet there is little formal work to evaluate such claims. Given the structure of comparative advantage and differing tradeability of certain occupational outputs, where would educational funding best be spent? And if indeed improvements to education would improve workers' global competitiveness, what are the potential distributional consequences? This paper marks a first step to address these questions. To this end, we develop a continuum-by-continuum model in which trade and educational institutions interact to determine individuals' skill acquisition decisions, the distribution of income, and ultimately the welfare consequences of freer trade. Our framework offers a novel lens through which we then can compare several simple educational and trade policy schemes.

The modelling strategy we propose is in contrast to most existing work in the trade literature, which restricts human capital decisions to a binary framework.[1] In these earlier models, workers are typically either skilled or unskilled, and the opening of trade will induce either skill upgrading (in a country with comparative advantage in skill intensive products) or skill downgrading (in countries with comparative advantage in 'basic' goods) through customary Stolper-Samuelson forces, but not both. A key prediction stemming from these models is immediate and unambiguous: in developed countries (which presumably hold comparative advantage in skill intensive goods), freer trade will induce workers to move to high skill, export-oriented sectors. The opposite holds true in countries with comparative advantage in less skill intensive goods (e.g. most developing countries), where freer trade would increase the relative demand for unskilled workers and and reduce workers' incentive to acquire human

[1]Recent exceptions are Yeaple (2005), Helpman, Isthoki, and Redding (2008), Anderson (2009), and Costinot and Vogel (2008), who also allow a continuous support of possible skill levels, though human capital is taken to be exogenous in all of these models.

capital. The obvious policy implication then is that educational subsidies have the potential to smooth workers' transition to a liberalized trade environment in rich countries, but at the same time would make adjustment to trade even more difficult in the developing world.

Although analytically parsimonious, such a simple binary approach clearly oversimplifies the process of skill acquisition. Not only are the implications for education policy implausibly stark, the economic prediction denies an important empirical regularity: that *within* countries freer trade causes some workers to 'sort down' – often into low-skill service sector jobs – while others simultaneously 'sort up' into higher skill jobs. Surely there are workers in rich countries who choose to acquire less human capital following trade liberalization, just as there are individuals in developing countries who acquire greater skill sets when trade barriers are lowered. Indeed, a series of recent studies has found evidence of exactly this phenomenon. Figure 1 reproduces a key finding by Goos and Manning (2007), who demonstrate that employment growth in the United Kingdom over the last few decades has been non-monotonic, with the low and top-end segments of the job quality spectrum experiencing considerable growth while employment in mid-range sectors has fallen. Similar evidence for the U.S. and Portugal has been demonstrated by Autor, Levy, and Murnane (2003) and Falvay, Greenaway, and Silva (2008), respectively.[2]

We propose a new modelling approach that captures the potential for observed non-monotonic shifts in job growth. Our framework features a continuum of heterogeneous agents who choose among a continuum of occupational sectors (or tasks), each of which requires a unique set of skills for employment. Each occupation is in turn used in the production of a particular intermediate product or service. Work-

[2]Also noteworthy is recent work by Ravallion (2009), who documents a converse shift in the developing world where the middle classes are expanding. See also the recent special report in The Economist, "Burgeoning Bourgeoisie: A special Report on the New Middle Classes in Emerging Markets," February 14, 2009.

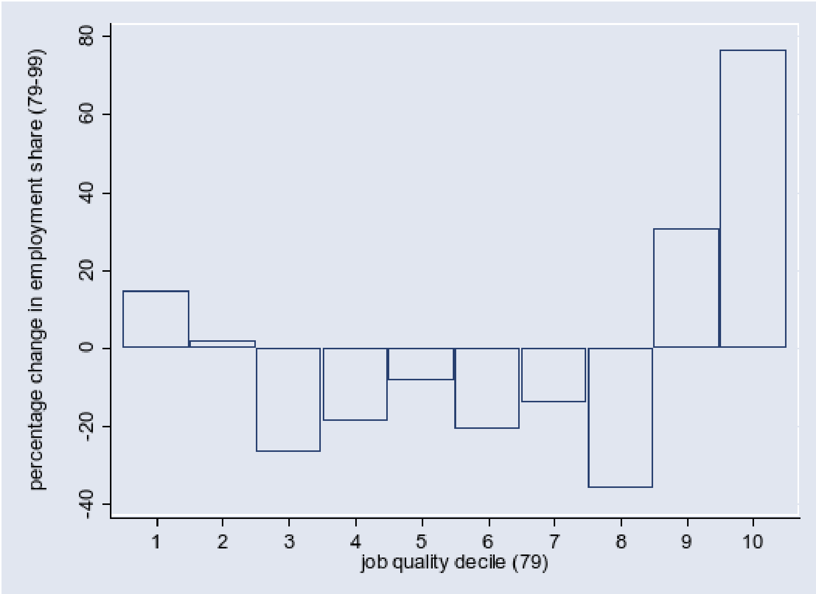

Figure 1: Non-Monotonic Changes in Employment (Goos and Manning (2007))

ers' wages are determined by sectoral technology and intermediate good/task prices — and thus indirectly by trade policy — while the cost of human capital acquisition is determined by individual characteristics and the country-specific structure of educational institutions and policies. Faced with the resulting incentive structure, agents of different inherent ability levels self-select into sectors by investing in the corresponding human capital.

Comparative advantage is driven by international differences in educational institutions and the resulting differential costs of skill acquisition. Trade liberalization leads to a remapping of agents to sectors, as would changes in trading partners' technologies, trade costs, or educational institutions that influence the cost of skill acquisition. The resulting shift in the demographics of skill composition can take many forms. One plausible and particularly salient scenario in line with the earlier diagram is the crowding out of the middle class towards the skill acquisition extremes, which could be brought about by increasing foreign competition in mid-technology intermediate goods/tasks. Following increased competition from foreign producers in these mid-range sectors, some formerly moderate ability agents would invest more in human capital, while others optimally would invest less. Thus, while the aggregate gains from trade are positive, the distributional consequences can be dramatic.

Turning to policy analysis, the model sheds light on the potential differential impact of strengthening educational institutions versus trade protection. In general, government subsidies to education or similar institutional improvements that decrease the cost of skill acquisition will influence workers' human capital decisions and thus the pattern of comparative advantage and trade. Targeted trade protection can have a similar effect by buoying the wages in import-sensitive sectors relative to the rest of the economy, but the concomitant consumption distortions imposed by tariffs make education policy a more efficient policy tool for redistributing income and human capital. Finally, we find that if the government has an explicit preference for a "strong middle class" then uniform educational reform will serve little purpose. Instead, educational incentives would need to induce skill upgrading among the least able workers through aggressive subsidization and reform at the most basic levels, and skill downgrading (or at least slower upgrading) among the educated elite. Naturally, such a policy would necessarily channel workers into the import-competing "middle class" sectors, and thus may prove an untenable long term solution.

Our approach in this paper is motivated in part by recent work in labor economics that analyzes the diverging development of different segments of the labor market. As noted earlier, Autor, Levy, and Murnane (2003), Goos and Manning (2007), and Falvay, Greenaway, and Silva (2008) document that employment growth has been non-monotonic across sectors in that it is positive at the low and high ends of the labor market, but negative in the middle. While Autor and Dorn (2007) propose a simple three-sector model to explain this divergence, we provide a theoretical framework that is considerably more general and links the shift in employment and human capital to trade liberalization. The second motivation of our work, to which we alluded earlier, is the two-sector limitation of most of the trade literature that endogenizes human capital formation. This limitation (of which we too are guilty, in Blanchard and Willmann (2008)) leads to the artificial result that trade liberalization goes hand in hand with uniform skill upgrading in developed economies and skill downgrading in emerging markets.

This paper also finds similarity with a number of important recent advances in trade theory. In modelling occupational output as tradeable tasks, we recall Grossman and Rossi-Hansberg (2006), who propose a two-sector model of trade in tasks and focus on the welfare effects of outsourcing those tasks that can be carried out abroad. Also similar is the recent work by Jung and Mercenier (2008), who propose a model that features outsourcing of intermediates at the same time as endogenizing the human capital decision. Their approach, however, is in many respects a two sector set-up; as a consequence, skill upgrading in their framework is necessarily uni-directional as in the more traditional trade literature. Finally, from a more technical standpoint, our continuum framework is reminiscent of Dornbusch, Fischer, and Samuelson (1977), Costinot and Vogel (2008), Anderson (2009), and Helpman, Isthoki, and Redding (2008)– all but the first of which can generate non-monotonic welfare consequences of trade – but none endogenize workers' human capital decisions or permit a role for educational institutions.

The remainder of the paper is structured as follows. In Section 2, we introduce the model, analyze the effects of trade under the small country assumption, and give the equilibrium conditions for the large country case. Section 3 assumes functional forms to examine further the inner workings of the model and to present the equilibrium characteristics of a two country case with non-monotonic skill change. Section 3.1 describes two important modeling extensions: first the potential for limited task diversification, and second the role of non-tradeable service sectors. Section 4 introduces the possibility of education subsidies and tariffs to explore further the model's policy implications. We conclude in Section 5.

2 The Model

The Home country is populated by a continuum of heterogeneous agents with unit mass. Individual agents differ in their inherent ability levels, a, assumed to be distributed continuously over the unit interval with cumulative distribution function $F(a)$ and corresponding density function $f(a)$. Home and Foreign have identical *ex ante* populations), although this assumption easily can be relaxed. Every agent is endowed with a single unit of labor, which is supplied inelastically to the labor market.

The economy produces a single homogeneous final good, Y, using constant returns to scale technology and a continuum of intermediate tasks (or products) $j \in [0,1]$, where j may be thought of as an index of the intermediate sectors' technological sophistication. Each intermediate sector uses a specialized type of labor and produces under constant returns and perfect competition. Productivity is assumed to be the same for all workers of an acquired skill type, regardless of the agent's inherent ability.[3] The final good serves as numeraire with price denoted by $p \equiv 1$. Finally, we choose units so that the real wage in sector j, measured in units of the final good Y, is simply the trading price of the relevant intermediate good/task and is denoted by $w(j)$.

In order to supply one unit of specialized labor of type j, agents have to acquire the required skills through training and education. The cost (in units of the numeraire,

[3] We could instead build worker heterogeneity into productivity (and thus wages) rather than education costs to generate the same sorting of workers across sectors, but we find the exposition somewhat cleaner with our formulation.

Y) to agent $a \in [0,1]$ of acquiring the skills for a given sector $j \in [0,1]$ is denoted by $c(j,a) \in C^2$. We assume that the cost of skill acquisition is increasing in the technological sophistication of the sector and decreasing in the ability level of the agent; further, the marginal cost of upgrading skills from one sector to the next is lower for high ability agents; finally, the cost of skill acquisition is convex across sectors for every agent. Formally, we make the following assumption on the cost of skill acquisition:

Assumption 1

$$\begin{aligned} \frac{\partial c(j,a)}{\partial j} &> 0, \qquad \frac{\partial c(j,a)}{\partial a} < 0 \\ \frac{\partial^2 c(j,a)}{\partial j \partial a} &< 0, \qquad \frac{\partial^2 c(j,a)}{\partial j^2} > 0. \end{aligned} \tag{2.1}$$

Additionally, in the interest of tractability, we will consider the following simplification:

$$c(j,a) \equiv h(a)g(j) \tag{2.2}$$

$$c^*(j,a) \equiv h(a)g^*(j), \tag{2.3}$$

where the functions $h(\cdot)$ and $g(\cdot)$ are twice continuously differentiable and non-negative over the unit interval. Note that Assumption 1 implies that $h'(a) < 0$, $g'(j) > 0$ and $g''(j) > 0$ for all $j \in [0,1]$.

Optimal Sorting and Production. Agents consume only the final good Y with non-satiated preferences. Thus, when deciding which sector to enter, every agent a chooses j to maximize his net real wage, $w(j) - c(j,a)$. Taking the wage schedule as given, the first order condition for each individual's optimal human-capital level is then:

$$\frac{\partial c(j,a)}{\partial j} = \frac{dw(j)}{dj}, \tag{2.4}$$

Using superscript dots to denote derivatives with respect to j, the first order condition for agent a's optimal human capital decision/sectoral choice may be rewritten as:

$$\dot{c}(j,a) = \dot{w}(j) = \dot{g}(j)h(a). \tag{2.5}$$

The second order condition of the agents' maximization problem is:

$$\ddot{w}(j) \leq \ddot{c}(j,a); \tag{2.6}$$

that is, the wage schedule — exogenous in a small open economy and endogenous if the country is large or autarkic — must be less convex than the cost function. Provided the second order condition is satisfied globally (i.e. for any $j \in [0,1]$), then $\dot{c}(j,a)$ crosses $\dot{w}(j)$ at most once from below for any given a, and (2.5) implicitly defines a unique critical value of j for each agent. The first order condition in (2.5) thus determines the following allocation (or self-sorting) of ability types to sectors:

$$a(j) \equiv h^{-1}\left(\frac{\dot{w}(j)}{\dot{g}(j)}\right). \tag{2.7}$$

Note that the second inequality in Assumption 1 ensures that $h(\cdot)$ is invertible so that $a(j)$ is well defined. When $a(j)$ is strictly monotonic, and thus invertible, we denote the mapping of ability types to sectors by $j(a)$.

The assumptions made earlier ensure several important properties of the mapping function in (2.7). The third inequality of Assumption 1 ensures single crossing, so that agents map to sectors assortatively;[4] the sorting function is strictly monotonic if the second order condition holds with strict inequality. Further, assuming that $h(\cdot)$ and $g(\cdot)$ are twice continuously differentiable implies that $a(j)$ is continuous (continuously differentiable), provided $w(j)$ is (twice) continuously differentiable. Finally, if the wage schedule is twice continuously differentiable, $a'(j)$ will be everywhere finite. Summarizing formally, we have the following:

Lemma 2.1 *For a given, strictly increasing wage schedule:*

[4]Note that $\dot{w}(j) > 0$ is a necessary condition for positive production in sector j.

i) $a(j)$ *is non-decreasing (strictly increasing) in* j *if* $\ddot{w}(j) \leq \ddot{c}(j,a)$ $(\ddot{w}(j) < \ddot{c}(j,a))$ $\forall j, a$.

ii) $a(j)$ *is continuous (continuously differentiable) in* j *if* $w(j) \in C^1(C^2)$.

iii) $a'(j)$ *is finite for all* j *if* $w(j) \in C^2$.

Proof: See appendix.

Notice that the derivative of the mapping function, $a'(j)$, indicates the density of agents in any given sector j. Zero employment in a range of sectors would be represented by a flat segment in the $a(j)$ mapping function. Part *i)* of the preceding lemma rules out the possibility of empty sectors if the wage schedule is everywhere less convex than the cost schedule for all agents and all sectors. By similar logic, a potential mass point in the distribution of agents across sectors would be represented by an infinite derivative of the $a(j)$ function. Part *iii)* indicates that continuity in the $\ddot{w}(j)$ schedule is sufficient to rule out the possibility of mass points, thus ensuring that the mapping of agents to sectors is strictly assortive: higher ability agents self select into *strictly* higher j occupations.

The sorting mechanism can best be illustrated by a pair of simple graphs. Figure 2 depicts the optimal sorting of agents to sectors as a function of the local wage schedule and the cost of education for each agent. We suppose for the sake of simplicity that the wage schedule is such that for every a there exists a unique optimal sectoral choice j, as depicted.[5] Panel A illustrates agents' optimal sectoral choice according to the first order condition in (2.5). Notice that the second order condition requires that $\dot{w}(j)$ crosses $\dot{c}(a,j)$ from above at the optimal occupation, $j(a)$. Further, the assumption that $\frac{\partial \dot{c}(a,j)}{\partial a} < 0$ ensures assortive matching of agents to sectors so that

[5]In principle, the derivative wage schedule could cross a given derivative cost function several times. In that instance, the globally optimal $j(a)$ would be an element (or subset) of the critical values defined by (2.7) for which the second order condition is also satisfied.

$j'(a) \geq 0$. Panel B depicts the resulting mapping of agents to occupational sectors.

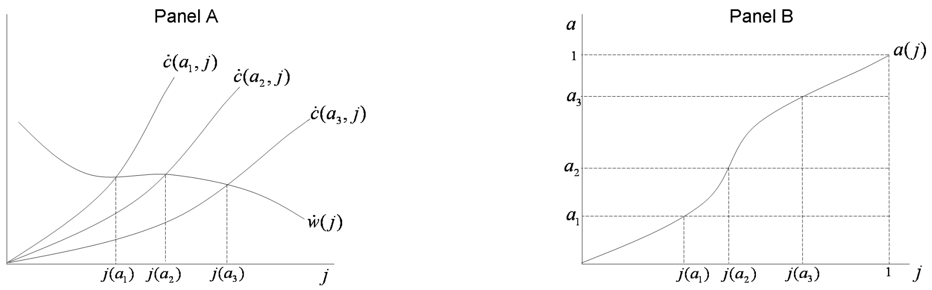

Figure 2: Optimal Sorting

To elucidate further the sorting mechanism, consider the comparative statics of an exogenous change in the domestic price/wage schedule. From Panel A, if the intermediate price schedule is everywhere flatter than before, so that $\dot{w}(j)$ shifts down, every agent will choose a lower j occupation, or 'sort down' in response to the decreased wage premia for skill upgrading.[6] Conversely, if the wage schedule is everywhere steeper than before, agents will choose higher skilled occupations, or 'sort up'. Note that for any vertical shift in the wage schedule for which $\dot{w}(j)$ remains unchanged, there will be no impact on agents' occupational choices or aggregate output. The following proposition formalizes this argument, assuming that the second order condition holds with strict inequality:

Proposition 2.1 *If the second order condition* (2.6) *holds with strict inequality, then for any two wage schedules* $w_1(j)$ *and* $w_2(j)$*:*

[6] In a dynamic framework in which agents cannot recoup the costs of over-education (in essence reselling their degrees), we would expect agents to remain in their same jobs or, if $\dot{w}(j) < 0$ in the relevant region, to shift into lower-skill work for which they are then overqualified.

i) if $\dot{w}_1(j_1(a)) = \dot{w}_2(j_1(a))$*, then agent a chooses the same sector under both price schedules,* $j_1(a) = j_2(a)$*;*

ii) if $\dot{w}_1(j_1(a)) > \dot{w}_2(j_1(a))$*, then agent a chooses a lower sector under* w_2*,* $j_2(a) < j_1(a)$*;*

iii) if $\dot{w}_1(j_1(a)) < \dot{w}_2(j_1(a))$*, then agent a chooses a higher sector under* w_2*,* $j_2(a) > j_1(a)$*.*

Proof: The result is immediate from the definition of $a(j)$ in (2.7).

Since we are interested in non-monotonic skill change, that is simultaneous 'sorting up' and 'sorting down', note that the proposition directly implies the following corollary:

Corollary 2.2 *Non-monotonic skill change occurs if and only if there exist* j_l *and* $j_h \in (0,1)$ *such that* $\dot{w}_1(j_l) < \dot{w}_2(j_l)$ *and* $\dot{w}_1(j_h) > \dot{w}_2(j_h)$ *(assuming that the second order condition holds with strict inequality).*

Non-monotonic skill change thus occurs if the two wage schedules cross at least once (and possibly more often).

Now that we have analyzed the sorting mechanism, we are in a position to close the model by defining appropriate equilibrium conditions. We will do so first for the case of a small open economy, before turning attention to the case of two large countries.

Equilibrium Conditions for a Small Open Economy.

The domestic supply schedule for intermediates is given by the density of workers of each type in the population, multiplied by the density of workers in each sector:

$$y_s(j) = a'(j)f(a(j)). \tag{2.8}$$

Notice that the supply of intermediates depends implicitly on the wage schedule and on the cost of skill acquisition through the $a(j)$ mapping function.[7]

Based on the total supply of intermediates, domestic output of the final good is given by $Y \equiv \psi(\vec{y})$, where $\psi(\cdot)$ denotes the constant returns technology used to produce the final good and each $y(j) \equiv y_s(j) + y_t(j)$ includes net imports of intermediate products, $y_t(j)$ (where negative $y_t(j)$ represent exports). We denote the unit factor demand for output of sector j by $x(j) \equiv x_j(\vec{w}, 1)$,[8] and note that in general it depends on the complete wage schedule.

The equilibrium conditions are then as follows. Full employment requires that the density of agents in each sector maps to the unit mass of population; i.e.:

$$\int_0^1 a'(j) f(a(j)) dj = 1. \tag{2.9}$$

Market clearing in each market for intermediates implies:

$$y(j) = a'(j) f(a, j) + y_t(j) = x(j) Y \qquad \forall j \in [0, 1]. \tag{2.10}$$

The zero profit condition in aggregate good production requires total revenue to equal total factor payments so that:

$$Y = \psi(\vec{y}) = \int_0^1 w(j)[a'(j) f(a(j)) + y_t(j)] dj. \tag{2.11}$$

Finally consumers' balanced budget condition requires that their consumption expenditure on the final good equals their net income:[9]

$$Y^c = \int_0^1 [w(j(a)) - c(a, j(a))] da, \tag{2.12}$$

[7]The function $a(j)$ also depends on production technology. Since technology differences are the main focus of Dornbusch, Fischer, and Samuelson (1977), we silence that mechanism in the baseline version of our model.

[8]Recall that constant returns to scale technology implies that conditional factor demand may be written $x_j^T(\vec{w}, Y) = x_j(\vec{w}, 1) Y \equiv \arg\min_{x_j} \vec{w} \cdot \vec{x}$ s.t. $\psi(\vec{x}) \geq Y$.

[9]Tariff revenue, if appropriate, would simply be added to the RHS of (2.12).

where Y^c denotes aggregate consumption of the final good.

For a small open economy, the system described by (2.9) - (2.12) pins down, for a given wage schedule, the equilibrium allocation of agents to occupational sectors, intermediate production levels and trade, as well as aggregate final good output and consumption. For a large economy, on the other hand, the wage/intermediate price schedule is endogenous and depends on the rest of the world.

Equilibrium Conditions for Large Economies.

The wage schedule becomes endogenous once we consider two large countries, say Home (the country previously described) and Foreign. In all relevant aspects but one let Foreign mirror Home: it has the same unit mass of population with an identical ability distribution, the same non-satiated preferences and inelastic labor supply, and the same production technology for intermediates and the final good. The one dimension along which we let Home and Foreign differ is the educational cost structure. We restrict attention to differences in educational institutions because this is the novel force driving comparative advantage and ultimately trade that it the focus of this paper.[10]

Under autarky each economy is characterized by a system of equilibrium conditions analogous to equations (2.9) - (2.12) above. Solving these, together with the autarky condition that trade is zero ($\vec{y}_t = \vec{y}_t^* = \vec{0}$) yields the autarkic equilibrium wage schedules, which we denote by $w_A(j)$ for Home and $w_A^*(j)$ for Foreign.[11] The equilibrium wage schedules in turn pin down all other variables of interest. Note that the first order condition ensures that the more convex (in j) is the local educational cost function, the steeper the autarkic equilibrium wage schedule (all else equal). This

[10]Since technology differences are the main focus of the seminal work by Dornbusch, Fischer, and Samuelson (1977), we silence that well understood mechanism in the baseline version of our model. One could easily introduce different intermediate good productivity in an extension of the basic model to study the interaction between technology, educational institutions, and trade.

[11]Note that we denote foreign country variables with an asterisk, as is standard.

makes sense; to the extent that skill upgrading becomes increasingly expensive for more sophisticated (high j) sectors, the higher the incremental wage increases must be to induce workers to enter the most demanding occupations.

Under free trade, intermediates can be traded on the world market. The set of world market clearing conditions for intermediates is therefore:

$$y(j) + y^*(j) = x(j)Y + x^*(j)Y^* \qquad \forall j \in [0,1]. \tag{2.13}$$

Additionally, we require that:

$$y_t(j) = -y_t^*(j) \qquad \forall j \in [0,1]. \tag{2.14}$$

That is, Home imports have to equal Foreign exports and vice-versa – the so-called NTWM (no trade with Mars) condition. The remaining equilibrium conditions – full employment, zero profit, and balanced budget in (2.9), (2.11), and (2.12) respectively – are the same as before, for both Home and Foreign. Together, these equilibrium conditions jointly determine one unified free trade equilibrium wage schedule, which we denote by w_{FT}. The equilibrium free trade wage schedule then pins down the supply of intermediates, aggregate output, and consumption in each country, as well as the pattern of trade. Note that for any sectors in which only one country produces, the market clearing condition in (2.13) has a single term on the left hand side, so that only the producing country's educational cost structure will influence the wage directly.

In general, the market clearing conditions will be characterized by a third order differential equation of the wage schedule over j.[12] Collapsing the system of intermediate market clearing conditions (of which there are an uncountable infinity) to

[12] $y(j)$ depends on $a'(j)$, which is a function of $\dot{w}(j)$ and $\ddot{w}(j)$ while $x(j)$ depends in general on the complete $w(j)$ schedule. Moreover, in a model with multiple final goods in which aggregate final goods output depends on its own price (i.e. $Y(p)$), the market clearing condition would instead be given by fourth order differential equation.

a single differential equation yields enormous returns in model tractability: namely, equilibrium properties can be summarized by the behavior of the wage schedule over $j \in [0,1]$ as above. At the same time, however, given that the equilibrium wage schedule is the solution to a differential equation of the third order, it should not be surprising that closed form solutions prove the exception rather than the rule.

The following section describes a case in which functional form assumptions offer clean analytical solutions in the general equilibrium model. In generating a set of closed form results, we highlight the role of educational institutions in determining both comparative advantage and the implications of trade for human capital acquisition, welfare, and income distribution within and across countries.

3 A General Equilibrium Example

In this section we provide a concrete example of our model that illustrates the simultaneous 'sorting up' and 'sorting down' of moderate ability agents and the negative welfare effects trade can have on the middle class. In order to make things tractable, we assume the following cost structures of education in home and foreign respectively:

$$c[j,a] = \frac{(1-a)}{a} * \frac{2j^2}{5} \tag{3.1}$$

$$c^*[j,a] = \frac{(1-a)}{a} * \frac{2j^3}{3} \tag{3.2}$$

On the factor demand side we assume Leontief production of the final good, thereby abstracting from possible substitution effects across intermediates.[13] With $\psi(\vec{y}) \equiv \min\{y_0, ..., y_1\}$, unit factor demand is simply one in each sector and country, regardless of the wage schedule; thus, $x(j) \equiv x_j(\vec{w}, 1) = x^*(j) = 1$. Following the solution

[13]More generally, substitution effects would dampen the magnitude of wage schedule changes, but would not overturn our qualitative findings; the technical benefit of the Leontief assumption is that $x_j(\vec{w}, 1) = 1 \forall j$ so that the intermediates market clearing condition is of only the second order (rather than third) and thus solvable.

procedure outlined in the previous section, we solve for the equilibrium wage schedules:

$$\dot{w}_A = \frac{4(1-j)}{5}, \tag{3.3}$$

$$\dot{w}_A^* = 2j - 2j^2, \tag{3.4}$$

$$\dot{w}_{FT} = \frac{j(2+j-10j^2) + \sqrt{j^2(4+j(4+4j(121+20j(-9+5j))))}}{10j}. \tag{3.5}$$

Where we have used the boundary condition that the wage schedule must be flat at the upper end, $\dot{w}(1) = 0$, to pin down the respective constants of integration.[14] Figure 3 shows that the slope of the equilibrium wage schedule under free trade is a weighted average of the autarky values. Note in particular that the intersection of the autarky wage slopes leads to the same value of the slope of the equilibrium wage schedule under free trade.

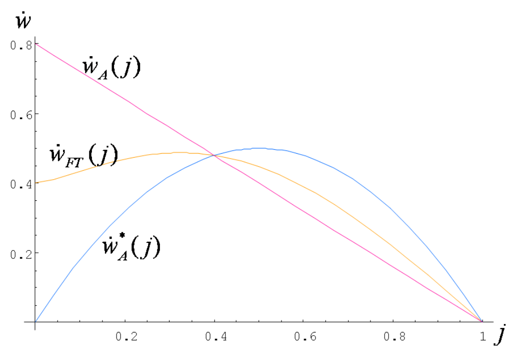

Figure 3: Slope of the Equilibrium Wage Schedules

As discussed before, the equilibrium wage schedule implies a corresponding mapping of agents to sectors by ability level. In autarky, of course, the Leontief

[14]This boundary condition ensures that there is not a mass of workers clustered in sector $j = 1$.

technology assumption implies a uniform density of workers across sectors; thus:

$$a_A(j) = a_A^*(j) = j. \tag{3.6}$$

Under free trade the sectoral mappings take a polynomial form (we omit the functional forms here for brevity), depicted in Figure 4. Home's free trade $a(j)$ mapping is in pink, Foreign's free trade mapping is in dark blue, and the autarkic mapping schedules for both Home and Foreign are represented by the light blue diagonal on the 45 degree line.

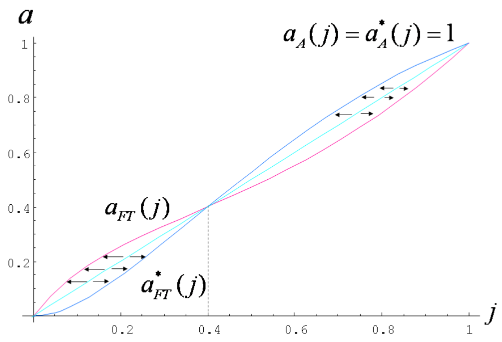

Figure 4: Mappings $a_{FT}(j)$ and $a_{FT}^*(j)$.

Figure 4 illustrates the reallocation of agents brought about by trade liberalization. Where the free trade mapping function lies above the diagonal, the corresponding ability level self-selects into a lower j sector following liberalization; i.e. agents have sorted down. Where the free trade mapping function lies below the forty-five degree line, agents self select into higher j occupations and human capital levels following the opening of trade. Overall, we see that in Home agents in the lower portion of the population distribution shift to lower j sectors, while agents above $a = .4$ shift up, thus vacating the middle j sectors toward the skill-acquisition extremes. The effects in Foreign are simply the reverse, since total labor supply of each factor must add to 2 under the Leontief final goods production structure.

Figure 5 depicts the resulting shift in employment density across sectors, which is equivalent to the supply of each intermediate output (given our assumption that $a \sim U[0,1]$). Again, note that the Leontief technology ensures uniform employment distribution in autarky.

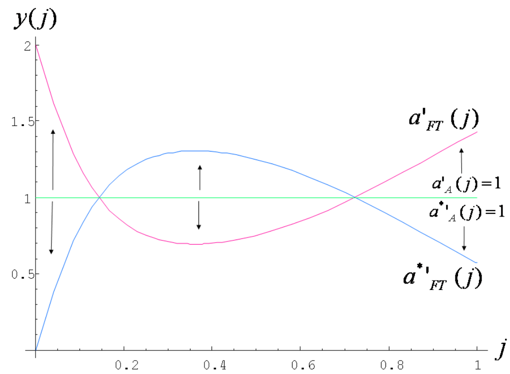

Figure 5: Employment Density by Sector: Autarky and Free Trade.

We again see that trade pushes Home workers to the skill acquisition extremes, while 'middle class' employment flourishes in Foreign post liberalization.

In addition to the employment and skill acquisition effects of trade liberalization, we are of course keenly interested in the welfare implications of freer trade. To get at real welfare effects, we first determine the real wage schedules under free trade and autarky using the zero profit condition for final goods production[15] to pin down the equilibrium wage level. The equilibrium real wage in a given sector $\hat{j}$ is given by $w_0 + \int_0^{\hat{j}} \dot{w}(j)dj$, where the base wage in sector $j = 0$ is determined by $w_0 \equiv$

[15]Under the Leontief production structure and choice of Y as numeraire, zero profit implies $1 = \int_0^1 w(j)dj$.

$1 - \int_0^1 \dot{w}(j)dj$. Solving, we find for low and high j sectors, the wage schedule increases at Home and decreases in Foreign country following trade liberalization, while the converse holds for mid-range sectors.

In what follows we first focus on the welfare effects in Home. As we will see below, the effects in Foreign are virtually a mirror image. The real welfare change consists of the effect on the real wage and the change in the realized cost of education. We analyze first the effect on the real wage. The two panels in Figure 6 depict respectively the change in the real wage in sector j and the change in the real wage of agent a given her optimal sectoral choice under each trading regime.

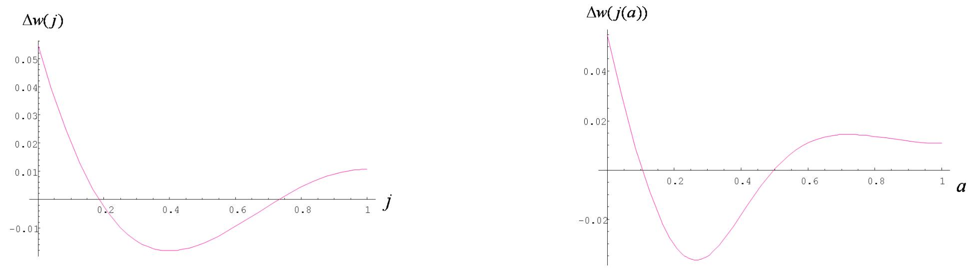

Figure 6: Effect of Trade on Wages at Home.

From the first panel in Figure 6, we see that real wages rise for the low and high j sectors, and fall for sectors j between about .2 and .7. The second panel takes into account the induced occupational shift, confirming that the change in realized real wages is non-monotonic across workers: agents with low ability earn higher real wages under trade, agents with high ability do as well, and agents in the lower-middle portion of the ability distribution see their real wages fall.

Figure 7 shows the change in the real cost of education across workers. Remembering that agents in the lower forty percent of the ability distribution optimally sort down while agents in the upper part of the distribution choose to sort up, it is obvious that the real cost of education should decrease for the left portion of the distribution and increase for the right.

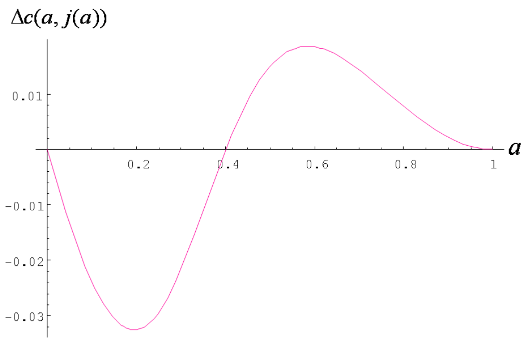

Figure 7: Change in the Home Real Cost of Education across Workers

Figure 8 shows the net welfare change for Home's population. Combined with the effect on the real wage, the adjustment for the changing cost of education has shifted the identity of the 'biggest loser' to the right. Indeed, consider the plight of the agent $a = .6$. Although her real wage has increased, the increased cost of education required to achieve the higher paying job more than offsets the wage gain so that the net welfare change is negative. Conversely, agent $a = .2$ suffers a substantial real wage loss yet enjoys a modest net welfare improvement due to his now lower cost of education. A crucial caveat to this second statement is that lower costs of education cannot be recovered if they are sunk. In a dynamic framework with unanticipated trade shocks, we therefore would expect to see the burden of increased costs of (potentially mid-career) education manifest in net welfare changes, while education savings would not be realized for the older generations.

Turning now to the Foreign country, we find that the net welfare effects are a mirror image of what happens at home. In contrast to Home, the real wage increases

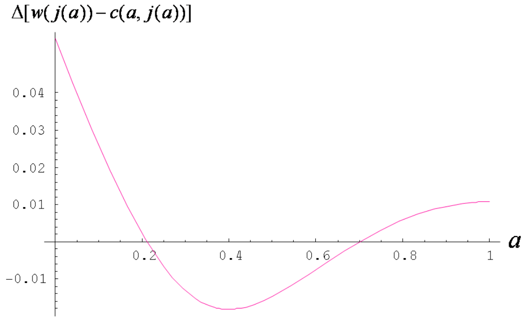

Figure 8: Net Welfare Effect of Trade in the Home Country.

most for Foreign agents in the middle of the income distribution while falling at the distribution extremes. At the same time, the cost of education rises for the lower forty percent of the distribution of workers and fall for the remainder. The net welfare effect of trade is shown in the final panel of Figure 9, where we see that middle ability agents gain from trade, while the highest and lowest ends of the population distribution lose.

To summarize the results from this general equilibrium example, we depict both the Home and Foreign net welfare changes by worker in Figure 9. While in the Home country it is the medium ability agents who suffer, their Foreign counterparts are the main beneficiaries of trade liberalization, together with high and low ability agents at Home. Integrating the net real effects in each country confirms that there are positive gains from trade for both countries.

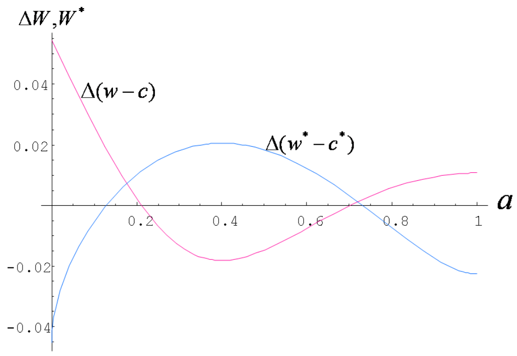

Figure 9: Opposing Individual Welfare Effect of Trade in Home and Foreign.

3.1 Extensions

3.1.1 Limited Diversification

In the baseline example, every task is carried out in both countries in equilibrium; that is, even under free trade, the production of intermediates remains fully diversified across the entire range of occupations. This needn't hold in general. We now present a modified case where trade induces one trading partner to stop production of a particular subset of intermediates, a phenomenon that is certainly relevant in reality, where for example certain inputs can be sourced locally only at very high cost. Suppose the educational costs take the following (slightly modified) form

$$c[j,a] = \frac{1}{a} * \frac{j^2}{2} \tag{3.7}$$

$$c^*[j,a] = \frac{1}{a} * \frac{2j^3}{3} \tag{3.8}$$

and assume the same production technology for the final good as before. We can solve for the autarky and free trade equilibrium wage schedules:

$$\dot{w}_A = 1, \tag{3.9}$$

$$\dot{w}_A^* = 2j, \tag{3.10}$$

$$\dot{w}_{FT} = \begin{cases} \frac{j+2j^2}{2j} & j < \frac{1}{4}(1+\sqrt{5}) \\ \frac{j}{2j-1} & j \geq \frac{1}{4}(1+\sqrt{5}) \end{cases} \tag{3.11}$$

These are graphically depicted in Figure 10. Note that the free trade equilibrium slope of the wage schedule (sandwiched between the autarky schedules) consists of two parts: up to $j = \frac{1}{4}(1+\sqrt{5}) \equiv \bar{j}$ both countries produce each task, whereas above only the home country does. This is because at the upper bound ($j = 1$) the marginal costs of education for the most able agents in both countries differ: the domestic agent with $a = 1$ only has half as high a marginal cost as the foreign agent, $a^* = 1$. Hence in equilibrium, the most able foreign agent does not find it worthwhile to acquire the sophisticated skills necessary to carry out the most sophisticated task. Instead she chooses $\bar{j} < 1$.[16]

As before, graphing the ability-to-task mappings under autarky and free trade in Figure 11 indicates how trade liberalization affects the skill acquisition decisions: to the left of the intersection agents at home sort down, whereas higher ability agents sort up and acquire more sophisticated skills, and the opposite happens in the foreign country. Thus the result that domestic agents vacate the middle obtains here as well.

Finally, comparing the wage change and the change in the cost of education gives the welfare effects depicted in Figure 12. Again we see that the middle ability

[16]We find this threshold by using the boundary condition $\dot{c}(a = 1, j = 1) = \dot{w}$ to determine the constant of integration of the w-dot schedule that equates home supply to world demand. Since 'smooth pasting' of the wage schedules for the upper and lower ranges implies the same derivative at the threshold, we find the threshold by plugging $a^* = 1$ into the 'only home production' schedule already determined. 'Smooth pasting' then pins down the constant of integration of the schedule to the left where both countries produce.

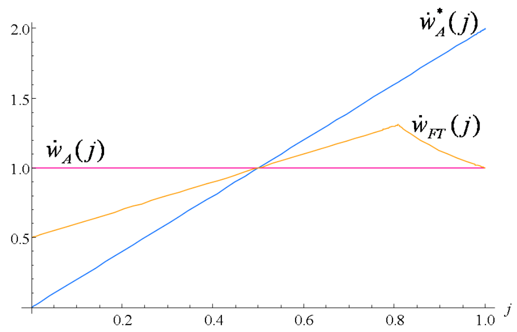

Figure 10: Wage schedules under autarky and free trade.

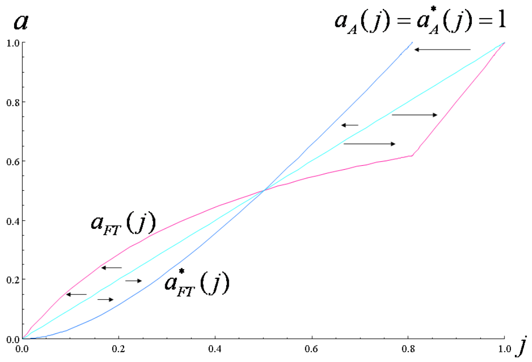

Figure 11: Ability-sector mappings under autarky and free trade.

agents at home lose out whereas agents at the bottom and top benefit from trade, and the welfare effects in the foreign country are the opposite. We have therefore confirmed that our previous result is robust to the possibility that countries restrict the range of tasks they produce in response to trade liberalization.

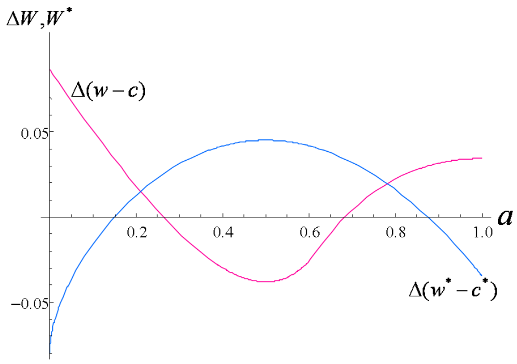

Figure 12: Net Welfare Effect of Trade in the Home Country.

3.1.2 Service Sectors (Non-traded goods)

4 Policy Implications

Our model immediately suggests itself to policy analysis. In the next section, we turn our focus to the potential role of educational policy and trade protection in shaping the distribution of income and human capital, noting in particular the extent to which the two policy instruments are complementary.

4.1 Educational Policy

In practice, educational policy takes many forms. Broadly speaking, educational initiatives can increase educational productivity (for instance, through curriculum reform or by expanding teacher education and incentives), decrease costs to students, or both. Here, we focus on the latter as it is simpler from a modelling perspective and effectively isomorphic in its underlying implication for human capital decisions.[17] We define an educational subsidy, $s(j)$, as a means to help individuals acquiring education in a given sector/task j shoulder the cost of doing so, $c(j, a)$. By assumption, the subsidy can vary by type of education or skill level to be acquired, but cannot be conditioned on the unobservable inherent ability level of the agent. To keep things simple, we assume subsidies are financed by a poll tax.

Recall the first order condition of the educational choice problem derived earlier in (2.5). Introducing an educational subsidy leads to the following augmented first order condition:

$$\dot{c}(j, a) - \dot{s}(j) = \dot{w}(j), \tag{4.1}$$

where $\dot{s}(j)$ is the first derivative of the subsidy function with respect to j (assuming $s(\cdot)$ is differentiable). We assume that the educational subsidy function is such that the left hand side of the augmented first order condition in (4.1) satisfies the derivative property assumptions (convexity, single crossing) we made regarding the educational cost function in (2.1).

For the following results, we adopt a small country assumption to rule out effects on the world price schedule. The following proposition follows directly from the preceding first order condition:

Proposition 4.1 *Under weak regularity conditions ($c(j, a) + s(j)$ satisfies derivative properties analogous to (2.1)), educational subsidies have the following local effects:*

i) if $\dot{s}(j) = 0$ the subsidy does not affect the educational decision of sector j agents,

ii) if $\dot{s}(j) > 0$ individuals initially in sector j choose higher education under the subsidy scheme,

[17] Improving the quality of education, rather than reducing the cost, would have the same effect on individuals' skill acquisition incentives, but would simultaneously increase the productivity per worker, thus decreasing the relative price of higher skill outputs when prices are endogenous.

iii) if $\dot{s}(j) < 0$ *individuals initially in sector* j *choose less education under the subsidy scheme.*

The first part of the proposition implies that a uniform subsidy does not affect individuals' educational choices.[18] Thus, while uniformly less expensive education would increase individuals' welfare, it will not encourage greater educational attainment unless the *marginal* cost of additional education falls. Parts *ii*) and *iii*) imply that if the subsidy scheme is uniformly progressive (in that $\dot{s} > 0 (< 0)$ for all j), all agents will sort into higher (lower) skilled occupations, apart from agent $a = 1$ ($a = 0$) who is already at the upper (lower) bound. More generally, the local effect of a subsidy change on agents initially located in a particular sector j will be to induce educational upgrading (downgrading) if the net marginal cost of education is decreased (increased) by the subsidy scheme.

Suppose then the government wants to use educational policy to soften the impact of globalization.[19] The negative impact of trade — at least in the example discussed earlier — is felt by middle-ability agents. If the government, perhaps in the interest of political stability that relies on a sizable middle class, wants to counteract the 'vanishing middle class' phenomenon, it has to provide an educational subsidy schedule that features a positive slope for low ability agents and a negative (or at least much flatter) slope for higher ability agents. In other words, it has to channel educational subsidies towards mid-level skill acquisition. The German educational system and the country's strong middle class can be regarded as anecdotal evidence for such an approach. In contrast, educational tax credits, which are roughly proportional to the cost of education, are regressive in the sense that they provide little to no (additional) subsidy through the secondary school level, and offer increasing subsidies

[18]This is little surprise as a uniform subsidy to all sectors simply cancels the poll tax (given full employment and positive education subsidies for all sectors $j \in [0, 1]$).

[19]We do not advocate such an objective from an efficiency point of view; rather, we simply explore the possibility as one that seems consistent with many politicians' stated goals.

thereafter.[20]

Targeted education subsidies toward middle class workers constitute a production subsidy to import-competing sectors. From an efficiency point of view, such a policy is clearly counter productive, and redistribution would be more efficiently achieved through more direct means. (Though of course more direct mechanisms are often politically infeasible.) If, however, the country is large enough to influence the world price schedule, then targeted middle class education subsidies would also improve the terms of trade and thereby shift part of the efficiency cost of middle class education subsidies onto foreign competitors. That said, a favorable shift in the terms-of-trade can be more efficiently achieved by trade policy than through educational subsidies, as we discuss in the next section.

4.2 Trade Policy

We open with a brief positive analysis of how trade policy influences educational choices, then move to a short normative discussion of optimal instrument choice. Let us initially adopt a small country assumption for simplicity, and define the specific tariff/export subsidy for good j by $t(j)$, so that the net domestic price/wage is given by $w^d(j) \equiv w(j) + t(j)$, where $w(j)$ now denotes the world price of good/task j.

On the supply side, the effect of a specific tariff (export subsidy) on human capital decisions is virtually identical to that of an educational subsidy. The isomorphism is readily apparent from the first order condition of the individuals' educational decision, which now takes the form:

$$\dot{c}(j, a) = \dot{w}(j) + \dot{t}(j),$$

[20]Though per-annum education credits are generally capped (making the effective subsidy uniform for sufficiently high tuition levels in a given year), the lifetime educational tax credit is clearly proportional to years of schooling, and thus (generally) increasing with skill acquisition.

where $\dot{t}(j)$ is derivative of the trade tax/subsidy schedule with respect to j (if the derivative exists). As with the education subsidy, a uniform trade tax schedule across all tasks will have no effect on agents' human capital decisions.[21] If the trade tax/subsidy schedule exhibits positive or negative slope, on the other hand, then it affects the individuals' skill acquisition decision: a positive (negative) slope induces individuals to choose a higher (lower) type of education. Given the immediate parallel with the case of an educational subsidy (which subtracts $\dot{s}(j)$ from the left hand side of the agents' first order condition rather than adding $\dot{t}(j)$ to the right hand side), we omit restating the results as a formal proposition. Simply note that the trade tax/subsidy result can be read directly from Proposition 4.1 replacing $s(j)$ with $t(j)$ and *educational subsidy* with *trade tax/subsidy.*

Now let us suppose that a policymaker seeks to mitigate the impact of globalization on the middle class by using tariffs to narrow the gap between autarky to free trade prices. To be consistent with commonly observed policy measures and WTO rules, we consider the effect of imposing tariffs on imported goods without introducing trade policy on the export side. Fundamentally, protectionist trade policy would 'close the lens' between autarkic and free trade wage schedules for the import-competing sectors, as is shown in figure ZZ. (More generally, we could interpret autarkic and free trade prices as world prices before and after the expansion or WTO accession of a major trading partner with comparative advantage in mid-range "middle class" goods.)

Notice that asymmetric trade policy to protect import-competing middle class sectors will necessarily shift the derivative net wage schedule, and will thus induce workers near the margins to exit export-oriented sectors in favor of import-competing sectors. More subtly, import tariffs would cause a non-concavity in the derivative wage schedule around each threshold separating import-competing and export goods.

[21] An ad-valorem tax/subsidy schedule would have an effect, of course, as its specific equivalent would imply a higher net wage derivative schedule for more skilled (higher wage) sectors.

For the simple case of an uniform import tariff, jumps in the derivative net wage schedule would occur at the thresholds, $\underline{j}$ and $\overline{j}$, as depicted in Figure 4.2. The derivative wage jumps occur at the borderline between imports and exports, and induce two 'empty' regions for which agents to the left (right) sort down (up). These 'empty' sectors have no domestic employment, and are thus Ricardian-like regions in which the Foreign country would be the sole producer worldwide, in line with the Dornbusch, Fischer, and Samuelson (1977) framework. Intuitively, by discontinuously increasing the relative return to import-competing sectors, tariffs would shift workers out of exporting industries. The extent to which export sectors are raided of their former workers depends on the degree of discontinuity in the *derivative* wage schedule following the imposition of tariffs. Dramatic increases in tariffs could thus have the unintended consequence of completely dismantling some previously export-competing industries.

It is clear that import protection would serve to shelter the middle class from global competition in mid-range sectors, and may thus serve a policymaker's (dubious) goal of manipulating human capital decisions and the pattern of employment. At the same time, however, tariffs also impose demand-side distortions that targeted education educational subsidies would not.[22] If the country is large, then the demand-side distortion would further improve the terms of trade (by decreasing world relative demand for mid-range goods), whereas in a small country, any demand-side distortion constitutes pure efficiency losses.

Moving to a normative comparison of different policy instruments, it is clear that the first best policy instrument depends on the government's underlying objective. If the government wants to shift the distribution of income, then of course lump sum taxes are the first best policy choice; if instead the goal is to shift the distribution of human capital, then educational subsidies (or taxes) are most efficient; and if the

[22]The exception is the case of Leontief final good production, in which case there is no distortion, but this is clearly razor's edge and unique to the Leontief functional form assumption.

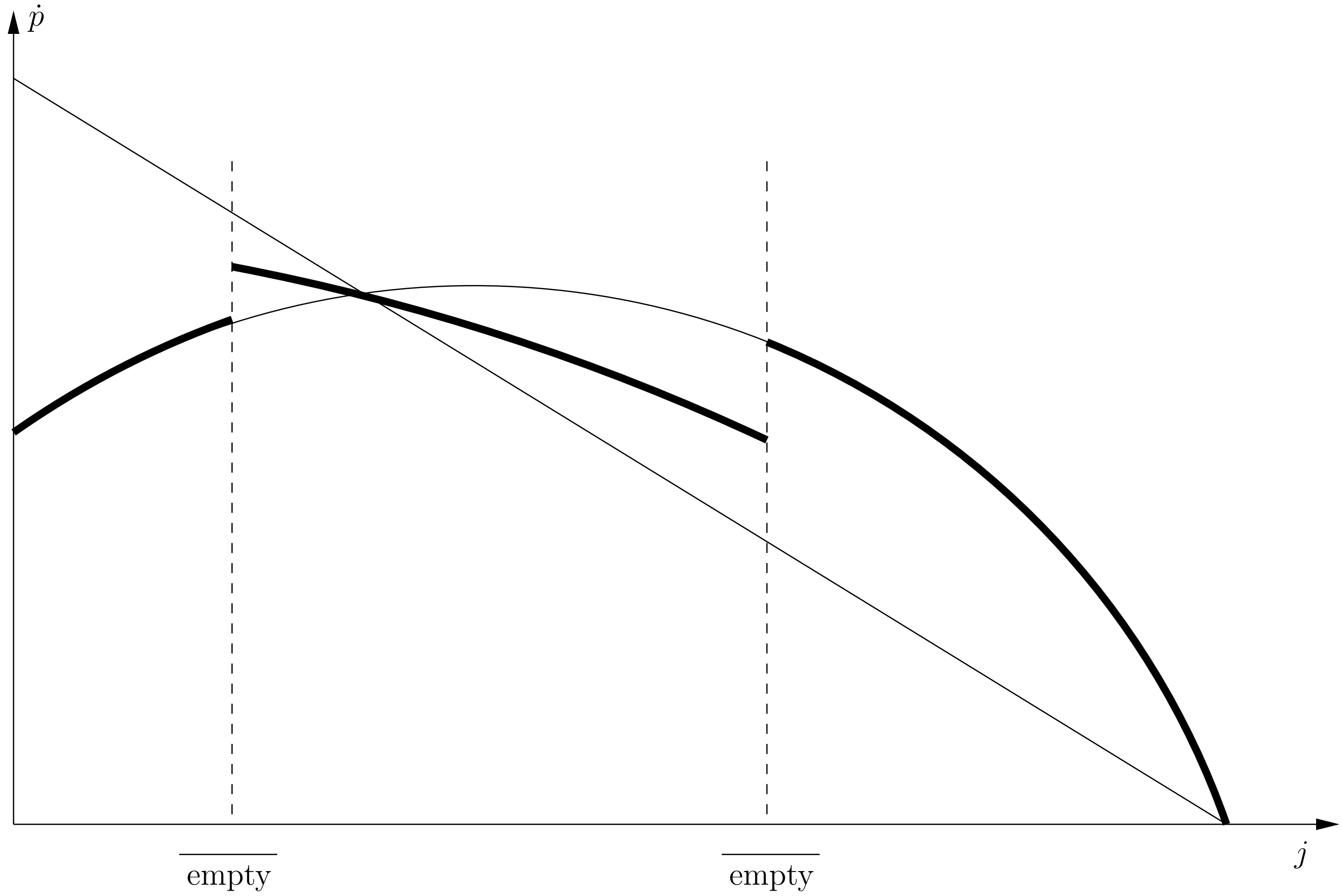

Figure 13: Effects of an import tariff

government simply seeks to manipulate the terms of trade, then of course trade policy remains the best means to achieve that end. Summarizing another way, education subsidies constitute a first best policy for shifting the distribution of human capital, a second best policy for shifting income (which is more efficiency achieved through direct redistribution and lump sum taxation), and a third best policy for manipulating world prices (which is better done via import tariffs). Trade taxes/subsidies, on the other hand, are first best for manipulating the terms of trade, second best for manipulating educational decisions (which is better achieved through education policy), and worst for shifting the distribution of income.

5 Conclusion

In this paper we develop a model of trade and education that allows for differentiated effects of trade liberalization on skill acquisition. Agents of different ability levels self-select into sectors by acquiring the specific education necessary to work in a particular sector, or perform a particular task. This mapping of ability level to sectors depends on the wage or price schedule and hence on a country's openness to trade. We show how changes in the price schedule affect this mapping and lead agents to sort up and down simultaneously. If a country's educational cost structure is less convex than that of its trading partners, then low ability agents sort down and higher ability agents sort up, and we obtain a 'vacating of the middle' with corresponding negative welfare effects for the middle class. This result provides one possible explanation for the current public concern over the negative effects of globalization on the middle class.

Our framework can shed light on the potential differential impacts of strengthening educational institutions. Government subsidies to education or similar institutional improvements that decrease the cost of skill acquisition over some ranges of sectors or for certain agents would impact the distribution of human capital decisions and thus the pattern of trade and comparative advantage, aggregate social welfare, and intra-national income distribution. In more general terms, the model developed here provides a novel reason for trade. By abstracting from differences in technology or preferences, we show how differences in educational institutions endogenously give rise to comparative advantage and hence trade.

In future work, we intend to use the framework developed here to analyze the effects of a differentiated educational policy that focuses on primary, secondary, or tertiary education. Uniform subsidies to education across the board are hardly the optimal policy recommendation resulting from our model. Perhaps highly targeted educational policies such as Brazil's are not as irrational and driven by specific interests as might seem at first sight. In addition, our model is well suited to study the effects of educational migration, i.e. the phenomenon recently documented in Blanchard, Bound, and Turner (2008) that students acquire education in another country and then either stay or return, which is of particular relevance for developing countries. In a somewhat more technical extension, we plan to explore systematically the nature of the interaction of technological changes with trade and education, in an effort to inform an empirical strategy for identifying the welfare effects of trade apart from technological innovation (while still recognizing the endogeneity of worker's human capital decisions). Finally, we intend a simplified version of this model as the building block for a dynamic endogenous trade policy model along the lines of our previous work in Blanchard and Willmann (2008).

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A1 Appendix

A1.1 Proof of Lemma 2.1

1. Taking the derivative of $a(j)$ with respect to j yields:

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)} \left[\frac{\ddot{w}\dot{g} - \dot{w}\ddot{g}}{\dot{g}^2} \right]. \tag{A1.1}$$

Substituting from the first order condition in (2.5):

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)} \left[\frac{\ddot{w}\dot{g} - h(a)\dot{g}\ddot{g}}{\dot{g}^2} \right]. \tag{A1.2}$$

Then, from the definition of the cost function:

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)} \left[\frac{\dot{g}(\ddot{w} - \ddot{c})}{\dot{g}^2} \right] \geq 0, \tag{A1.3}$$

using the second order condition ($\ddot{c} \geq \ddot{w}$) and the assumptions on the cost function in (2.1), which imply that $\dot{g} > 0$ and $h'(x) < 0$ iff $x > 0$. By assumption, $\dot{w} \geq 0 \forall j$, so

$$a'(j) = \underbrace{\frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)}}_{(-)} \left[\frac{\dot{g}(\ddot{w} - \ddot{c})}{\dot{g}^2} \right] > 0 \iff \ddot{c} > \ddot{w}. \diamond \tag{A1.4}$$

2. From the definition of $a(j)$:

$$a(j) = h^{-1}\left(\frac{\dot{w}}{\dot{g}}\right). \tag{A1.5}$$

Both $g(\cdot)$ and $h(\cdot)$ are twice continuously differentiable and invertible by assumption. Thus, $a(j)$ is continuous in j if $\dot{w}(j)$ is continuous (i.e. $w(j) \in C^1$). Moreover, $a(j)$ is continuously differentiable in j if $\ddot{w}(j)$ is continuous (i.e. $w(j) \in C^2$).

3. From part (1) above:

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)}\left[\frac{\ddot{w}\dot{g} - \dot{w}\ddot{g}}{\dot{g}^2}\right]. \tag{A1.6}$$

Again from our earlier assumptions, $h'(\cdot)$, $\dot{g}$, and $\ddot{g}$ are finite, and $\dot{g} > 0$. Thus, $a'(j) < \infty$ if and only if $\ddot{w}\dot{g} - \dot{w}\ddot{g} < \infty$. A sufficient condition is $w(j) \in C^2$, as stated in the lemma.